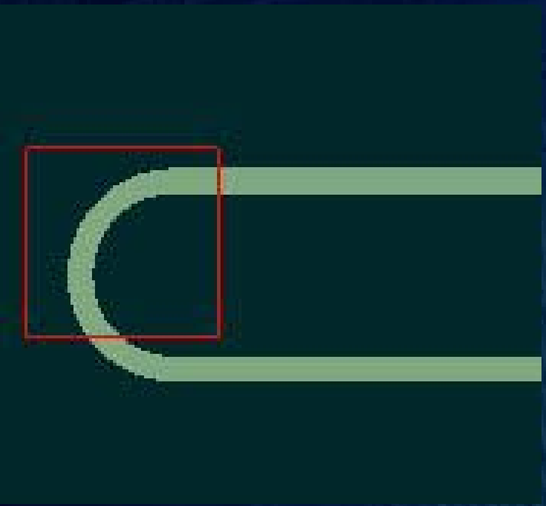
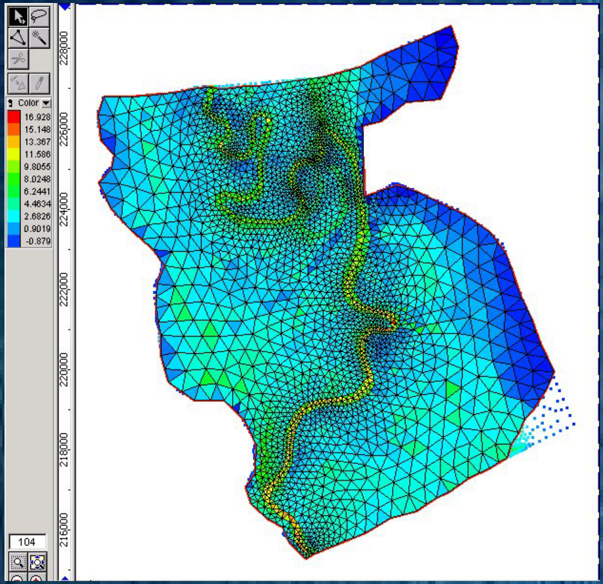
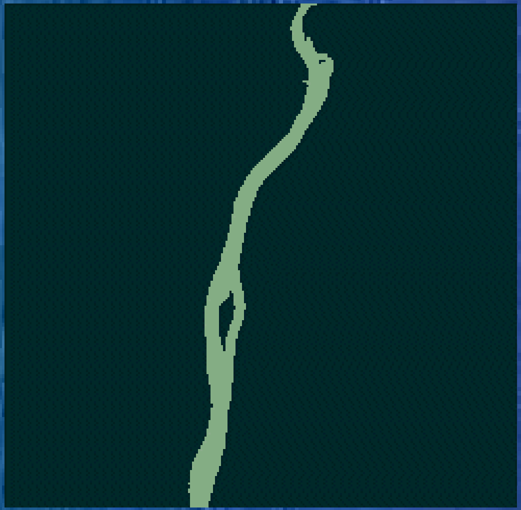
UnTRIM Modeling of Environmental Flows In Rivers, Lakes and Estuaries

An Introduction to UnTRIM Model







Ralph T. Cheng
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Outlines

I. View Points of Numerical Modeling

II. Numerical Methods and the TRIM family of models

III. Examples of UnTRIM Modeling of Environmental Flows in Rivers, Lakes, and Estuaries

Numerical Modeling of Environmental Flows

Scales: Physical Properties or Physical Processes

Spatial and Temporal

Scales

Need the <u>Right Model</u> to represent the proper physical properties and to resolve the physical processes of the environmental problem

10 Great Lies of CFD Codes – Los Alamos

(Food for thought in David Letterman Style)

Right Model?

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(Food for thought in David Letterman Style)

10. The technique was first developed here!

10 Great Lies of CFD Codes – Los Alamos

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9. You can run the code without the manual!
10. The technique was first developed here!

10 Great Lies of CFD Codes – Los Alamos

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8. There are no more bugs in the code, only undocumented features!

9. You can run the code without the manual!

10. The technique was first developed here!

10 Great Lies of CFD Codes – Los Alamos

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7. User Friendly, minimal learning curve!
8. There are no more bugs in the code, only undocumented features!
9. You can run the code without the manual!
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10 Great Lies of CFD Codes – Los Alamos

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6. All physics are compatible!
7. User Friendly, minimal learning curve!
8. There are no more bugs in the code, only undocumented features!
9. You can run the code without the manual!
10. The technique was first developed here!

10 Great Lies of CFD Codes – Los Alamos

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5. Robust and accurate!

10 Great Lies of CFD Codes – Los Alamos

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4. Executable on all machines without modifications!
5. Robust and accurate!

10 Great Lies of CFD Codes – Los Alamos

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3. Standardized graphics output, compatible with third party post-processors!
4. Executable on all machines without modifications!
5. Robust and accurate!

10 Great Lies of CFD Codes – Los Alamos

(Food for thought in David Letterman Style)

2. The manual has everything you need to run the code!
3. Standardized graphics output, compatible with third party post-processors!
4. Executable on all machines without modifications!
5. Robust and accurate!

10 Great Lies of CFD Codes – Los Alamos

(Food for thought in David Letterman Style)

1. It will solve your problem without modifications!
2. The manual has everything you need to run the code!
3. Standardized graphics output, compatible with third party post-processors!
4. Executable on all machines without modifications!
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10 Great Lies of CFD Codes – Los Alamos

(Food for thought in David Letterman Style)

Right Model?

Numerical Modeling

25 years Collaborations with Vincenzo Casulli

Cheng, R.T., and Casulli, V., 1982, On Lagrangian residual currents with application in South San Francisco Bay, CA, Water Resources Research, v. 18, No. 6, p. 1652-1662.

Numerical Modeling

25 years Collaborations with Vincenzo Casulli

The TRIM Family of Models From TRIM to UnTRIM

- Solution of Shallow Water Equations
- Transient, Multi-Dimensional (3D, 2D, 1D)
- Simultaneous Solution of Transport Variables
- Semi-implicit Finite-Difference Method
- Boundary Fitting Unstructured Grid Mesh

Formulating the Algorithm for a Numerical Model

Desirable Properties of a Numerical Model:

1. Stability
2. Accuracy (Require compromise)
3. Efficiency

Numerical Algorithm

From PDE to Discrete Algebraic System:

Spatial discretization:

Finite difference, Finite Element, Finite Volume

Temporal discretization:

Explicit scheme, Implicit scheme, Semi-implicit

Numerical Foundation of TRIM (Background)

Casulli, V., 1990, Semi-implicit Finite-difference Methods for the Two-dimensional Shallow Water Equations, J. Comput. Phys., V. 86, p. 56-74.

Desirable Properties of a Numerical Model:

1. Stability 2. Accuracy 3. Efficiency

(Compromise)

Stability Analysis: Gravity wave terms and velocities in Continuity Eq. control the numerical stability

Method of Solution:

1. Treat those terms implicitly, and the remaining terms explicitly.
2. Substituting momentum Eqs. into continuity Eq., resulting a matrix equation that determines the water surface of the entire domain.

2D Depth-Averaged Shallow Water Equations

Continuity Eq.:

$$\frac{\partial \varsigma}{\partial t} + \frac{\partial[(h+\varsigma)U]}{\partial x} + \frac{\partial[(h+\varsigma)V]}{\partial y} = 0$$

X-Momentum Eq.:

$$\frac{DU}{Dt} - fV = -g\frac{\partial \varsigma}{\partial x} + \frac{1}{\rho_o(h+\zeta)}(\tau_x^w - \tau_x^b) + A_h \nabla^2 \mathrm{U} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial x}$$

Y-Momentum Eq.:

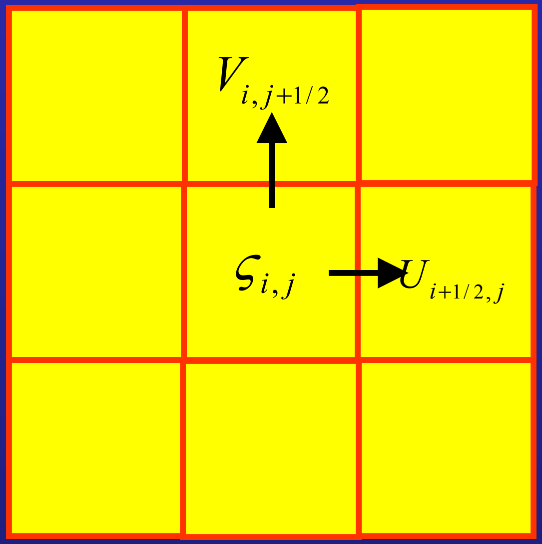
$$\frac{DV}{Dt} + fU = -g\frac{\partial \varsigma}{\partial y} + \frac{1}{\rho_o(h+\zeta)}(\tau_y^w - \tau_y^b) + A_h \nabla^2 \mathrm{V} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial y}$$

Eulerian-Lagrangian Method (ELM) => Stability (von Neumann)

X-Momentum Eq.:

$$\frac{DU}{Dt} - fV = -g\frac{\partial \varsigma}{\partial x} + \frac{1}{\rho_o(h+\zeta)}(\tau_x^w - \tau_x^b) + A_h \nabla^2 \mathrm{U} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial x}$$

Semi-implicit FD: Algebraic Eq. of $\varsigma_{i,j}^{n+1}, U_{i+1/2,j}^{n+1}, \varsigma_{i+1,j}^{n+1}$



Y-Momentum Eq.:

$$\frac{DV}{Dt} + fU = -g\frac{\partial \varsigma}{\partial y} + \frac{1}{\rho_o(h+\zeta)}(\tau_y^w - \tau_y^b) + A_h \nabla^2 \mathrm{V} - \frac{g}{2\rho_o}(h+\varsigma)\frac{\partial \rho}{\partial y}$$

Semi-implicit FD: Algebraic Eq. of $\varsigma_{i,j}^{n+1}, V_{i,j+1/2}^{n+1}, \varsigma_{i,j+1}^{n+1}$

Substituting the momemtum Equations into

Continuity Eq.: $$\frac{\partial \varsigma}{\partial t}+\frac{\partial[(h+\varsigma)U]}{\partial x}+\frac{\partial[(h+\varsigma)V]}{\partial y}=0$$

$$(1+A_{i+1,j}+B_{i-1,j}+C_{i,j+1}+D_{i,j-1})\varsigma_{i,j}^{n+1}$$
$$-A_{i+1,j}\varsigma_{i+1,j}^{n+1}-B_{i-1,j}\varsigma_{i-1,j}^{n+1}-C_{i,j+1}\varsigma_{i,j+1}^{n+1}-D_{i,j-1}\varsigma_{i,j-1}^{n+1}=E_{i,j}^{n}$$

With all coefficients are positive.

The governing matrix equation is symmetric, diagonally dominant, and positive definite. Numerical solution is achieved by a preconditioned conjugate gradient method.

Some Numerical Properties

- Convective terms- Eulerian-Lagrangian method
- Gravity wave terms - unconditionally stable
- Discretized equation - properly accounts for positive and zero depths
- Wetting and drying of cells are treated correctly
- TRIM2D successfully implemented to reproduce sharp hydrographs of riverine flows and for estuaries
- The model is robust and efficient

TRIM_2D: Extensive applications in San Francisco Bay

Cheng, R. T., V. Casulli, and J. W. Gartner, 1993, Tidal, residual, intertidal mudflat (TRIM) model and its applications to San Francisco Bay, California, Estuarine, Coastal, and Shelf Science, Vol. 36, p. 235-280.

What does TRIM model stand for?

TRIM stands for Tidal, Residual, Inter-tidal Mudflat

TRIM also implies simple and elegant in numerical algorithm and model code, a goal that we are striving for!

From TRIM Series of Models to UnTRIM

Systematic Development of TRIM Models:

TRIM_3D: Applications in San Francisco Bay and others

Casulli, V. and R. T. Cheng, 1992, Inter. J. for Numer. Methods in Fluids

Casulli, V. and E. Cattani, 1994, Comput. Math. Appl., Stability, accuracy and efficiency analysis of TRIM_3D, θ-method for time-difference

Cheng, R. T. and V. Casulli, 1996, Modeling the Periodic Stratification and Gravitational Circulation in San Francisco Bay, ECM-4.

TRIM_3D: Non-hydrostatic

Casulli, V. and G. S. Stelling, 1996, ECM-4

Casulli, V. and G. S. Stelling, 1998, ASCE, J. of Hydr. Eng

UnTRIM model:

Casulli, V. and P. Zanolli, 1998, A Three-dimensional Semi-implicit Algorithm for Environmental Flows on Unstructured Grids, Proc. of Conf. On Num. Methods for Fluid Dynamics, University of Oxford.

Extension to Unstructured Grid Model -- UnTRIM

TRIM Modeling Philosophy:

1. Semi-implicit Finite-Difference Methods
2. Θ-Method for time difference
3. Solutions in Physical Space, regular mesh, no coordinate transformations in x-, y-, or z-directions
4. In complicated domain, refine grid resolution if necessary
5. Pursue computational efficiency and robustness

UnTRIM (Unstructured Grid TRIM model) follows the SAME TRIM modeling philosophy, except the finite-difference cells are boundary fitting unstructured polygons!

Summary of the UnTRIM Model:

Governing equations (Hydrostatic Assumption)

Continuity and Free-surface Equations

$$Div(\vec{U}) = 0$$

Incompressibility

$$\frac{\partial\,\varsigma}{\partial\,t} + \nabla \bullet \left[\int_{-h}^{\zeta} \vec{V}\,dz\right] = 0$$

Free-surface equation

Horizontal Momentum Equation in $\vec{N}_j$ direction for velocity V_j

$$\frac{DV_j}{Dt} - f(\nabla\times \vec{V})\bullet \vec{N}_j = \frac{\partial}{\partial z}\left(\nu_{\mathbf{v}}\frac{\partial}{\partial z}V_j\right) + \nu_{\mathbf{h}}\nabla^2 V_j - g\frac{\partial\,\zeta}{\partial\,N_j} - \frac{g}{\rho_o}\frac{\partial}{\partial N_j}\int_z^{\zeta}(\rho-\rho_o)dz'$$

where $\nabla\times(\ )$ is cross product, $\nabla\bullet(\ )$ is inner product, $\nabla^2(\ )$ is the Laplacian, and $\vec{V}$ is the velocity in the horizontal plane.

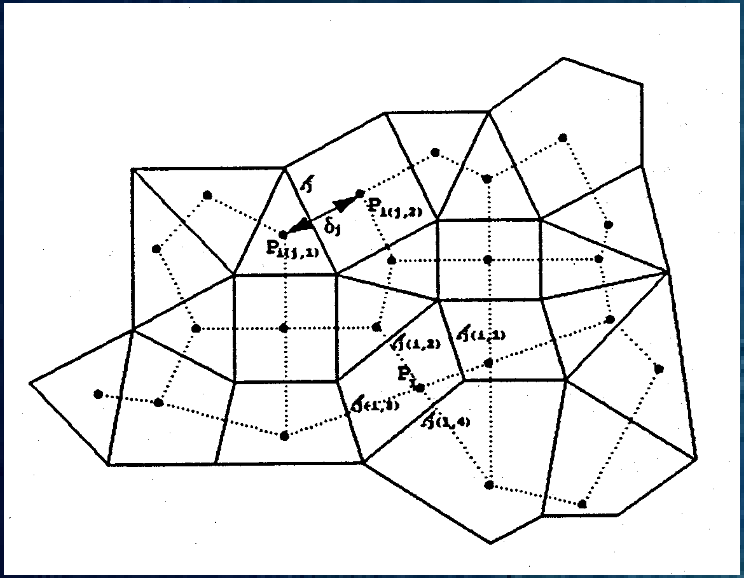
Transport Equations

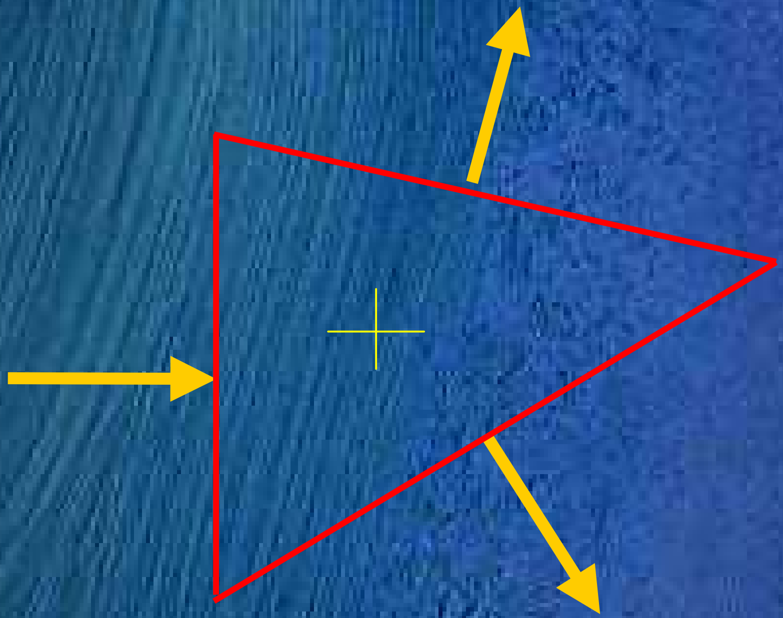
$$\frac{D}{Dt}\mathbf{C_j} = \frac{\partial}{\partial z}\left(\mathbf{K_v}\frac{\partial}{\partial z}\mathbf{C_j}\right) + \mathbf{K_h}\nabla^2\mathbf{C_j} \qquad \mathbf{j = 1, 2, 3, \ldots}$$

Lagged one time-step

And an equation of State

1. Semi-implicit finite-difference of momentum Eq. in the normal direction to each face is applied!
2. Applied the Finite-Volume integration of the free surface equation!
   Local and global conservation of volume is guaranteed!



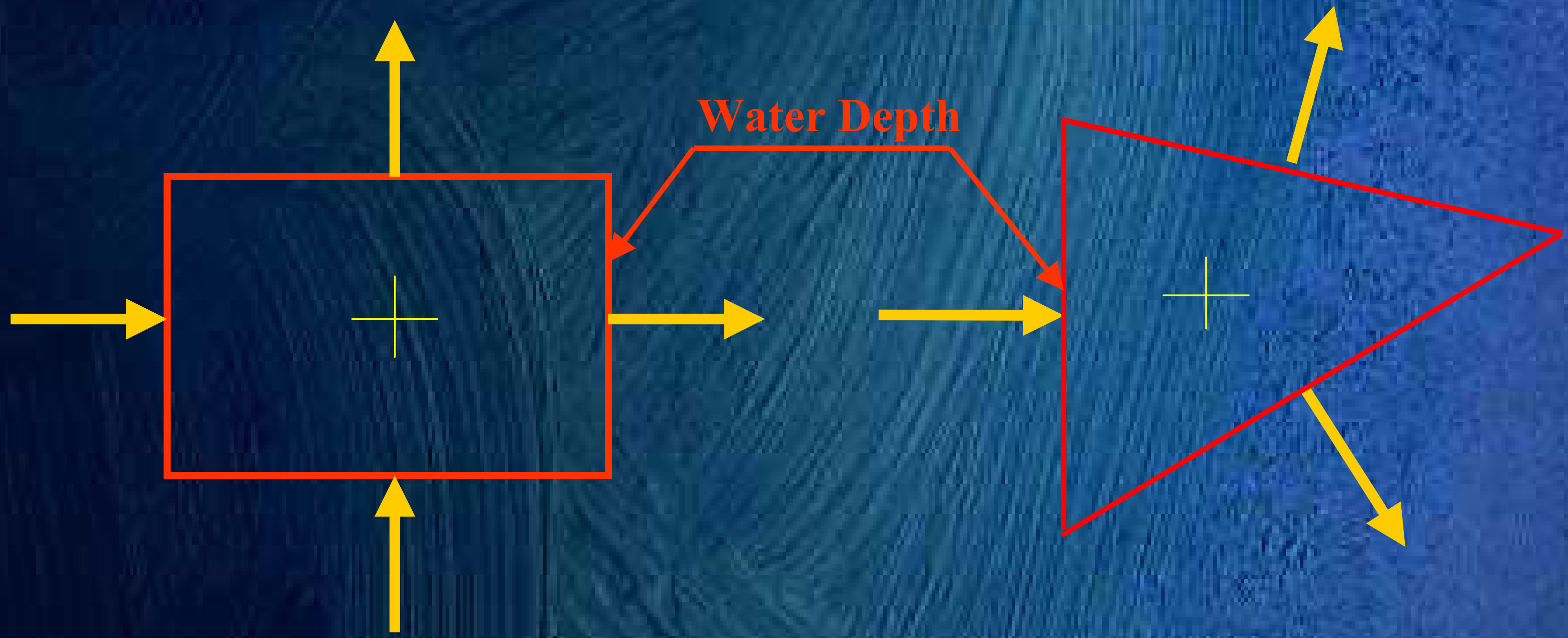


3. The resultant matrix equation determines the water surface elevation for the entire field.

1. **Semi-implicit finite-difference of momentum Eq. in the normal direction to each face is applied!**

2. **Applied the Finite-Volume integration of the free surface equation!**

   Local and global conservation of volume is guaranteed!



3. **The resultant matrix equation determines the water surface elevation for the entire field.**

Summary of Numerical Algorithm

Momentum Equation in $\vec{N}_j$ direction for velocity V_j relates

V_j and ζ (left) and ζ (right) on <u>each face</u> of a polygon

Continuity and Free-surface Equations

$$Div(\vec{U}) = 0$$

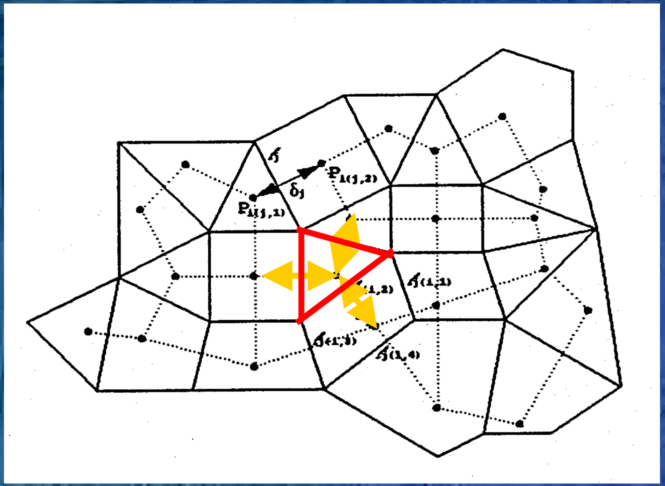
$$\frac{\partial \varsigma}{\partial t} + \nabla \bullet \left[\int_{-h}^{\varsigma} \vec{V}\, dz \right] = 0 \qquad \Rightarrow \qquad \frac{\partial \varsigma}{\partial t} + \oint \left(\int_{-h}^{\varsigma} \vec{V} dz \right) \bullet d\,\vec{s} = 0$$

Finite Volume integration over each polygon =>
V's are eliminated giving a Matrix Eq. for ζ

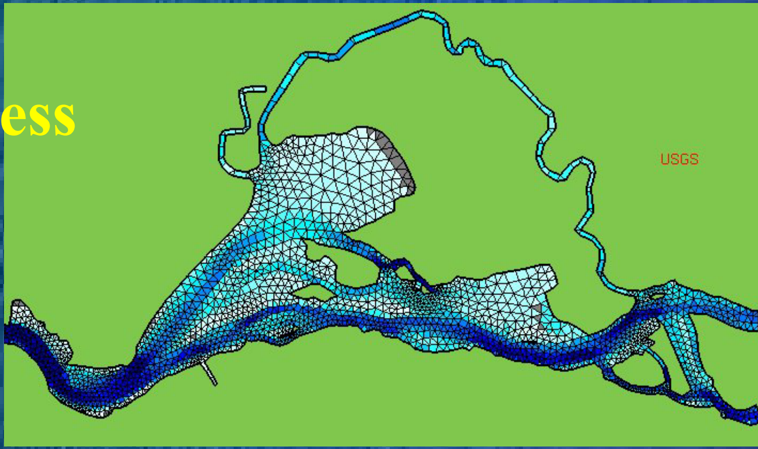
The continuity equation and the momentum equations are truly coupled in the solution. No mode splitting is used!

Issues of unstructured grids

User must define:

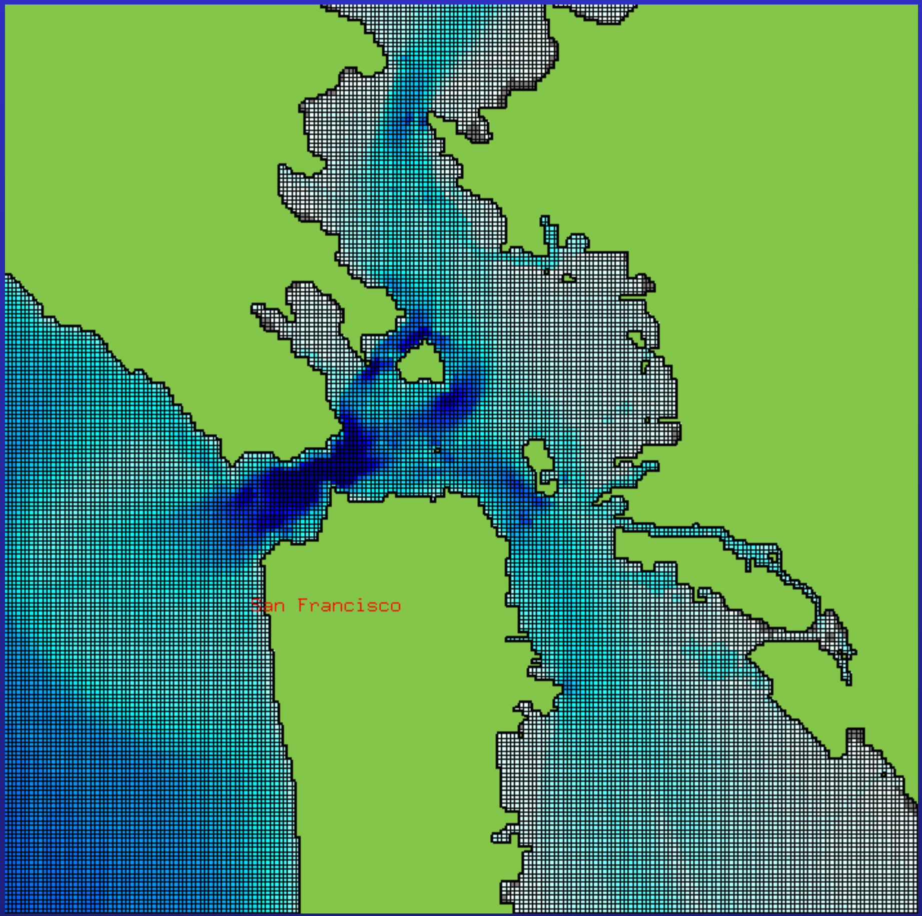


1. Number and locations of nodes
2. Polygon number and its relation with nodes (connectivity)
3. Each side is numbered, left and right polygons are defined (connectivity)
4. Center coordinates of each polygon
5. Vertical layers are of constant thickness (variable in z) except the bottom and free-surface; a stack of prisms
6. Water depth and normal velocity are defined on the sides
7. Water elevation is defined at the center of the polygon



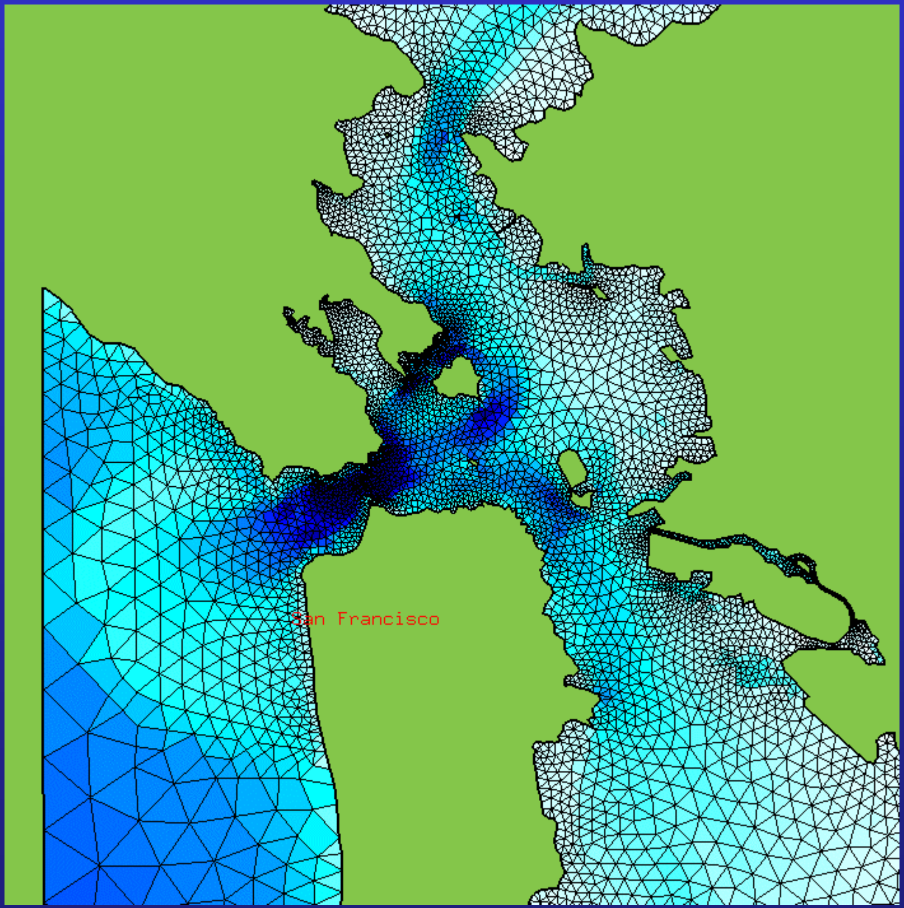
San Francisco Bay

(All Rectangles)



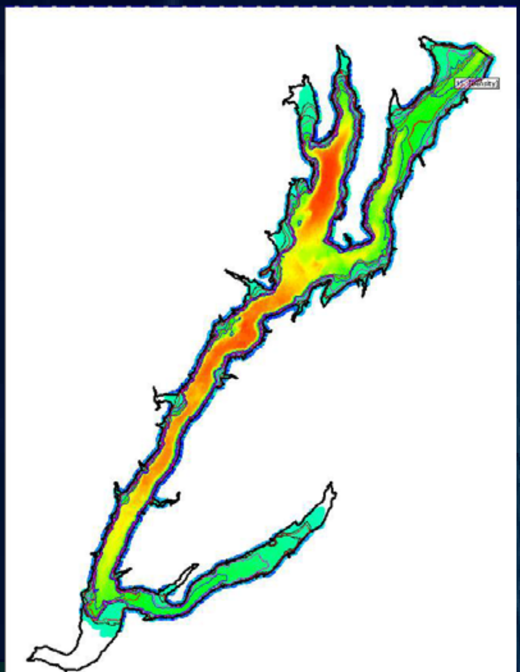
48506 nodes, 45841 polygons
94374 sides on the top layer
42 layers, 1,160 K faces, $\Delta t = 180$
(R= simulation/CPU = 25)
on 3.0 GHz PC

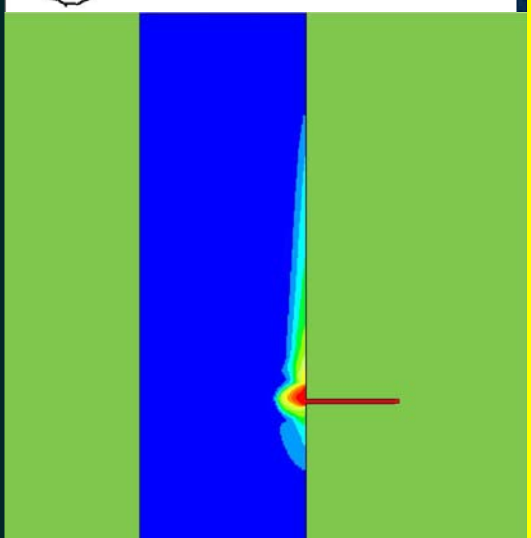
(Mixed Polygons)

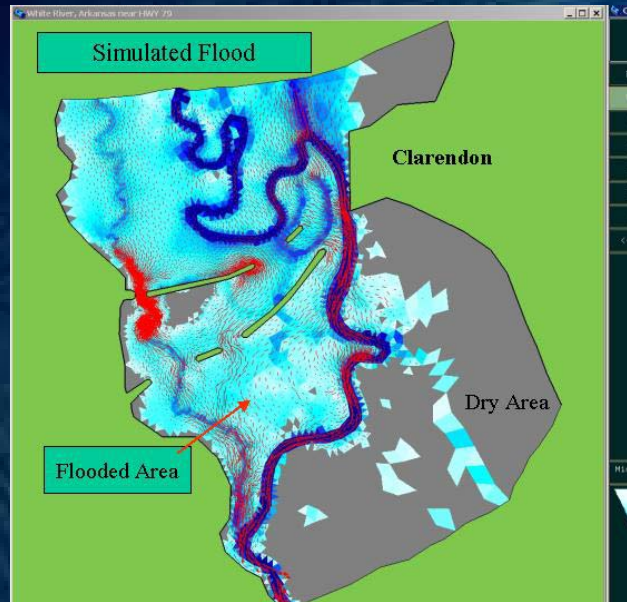


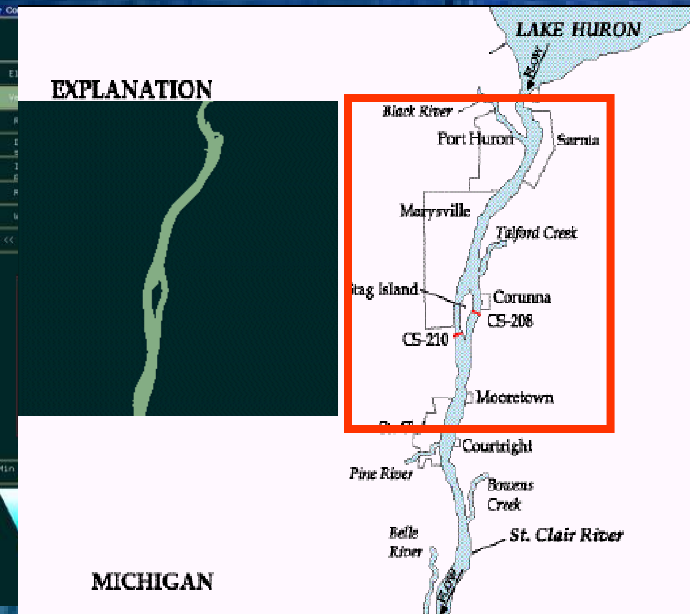
12682 nodes, 20126 polygons
32827 sides on the top layer
42 layers, 295 K faces, $\Delta t = 180$
(R= simulation/CPU = 100)
on 3.0 GHz PC

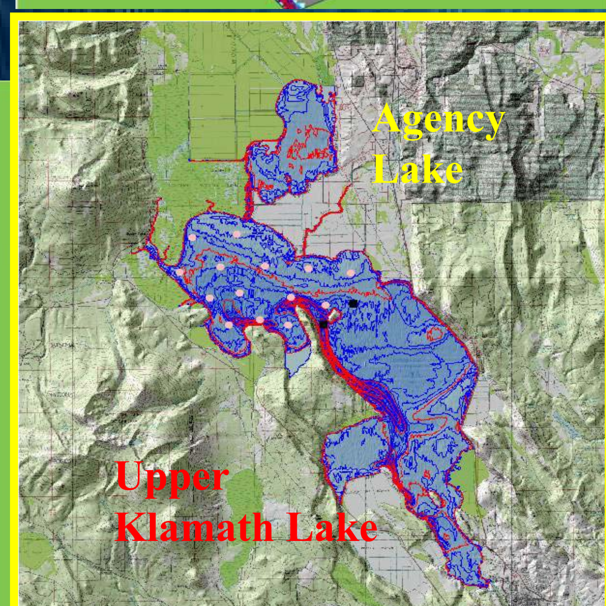
The UnTRIM Model has been applied to Rivers, Lakes and Estuaries

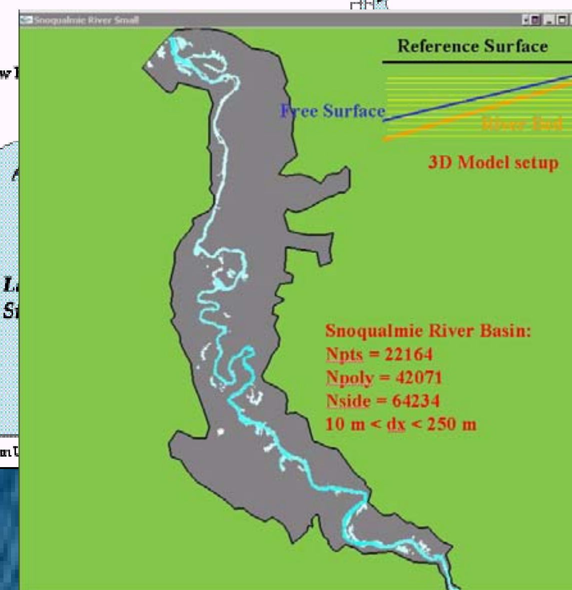




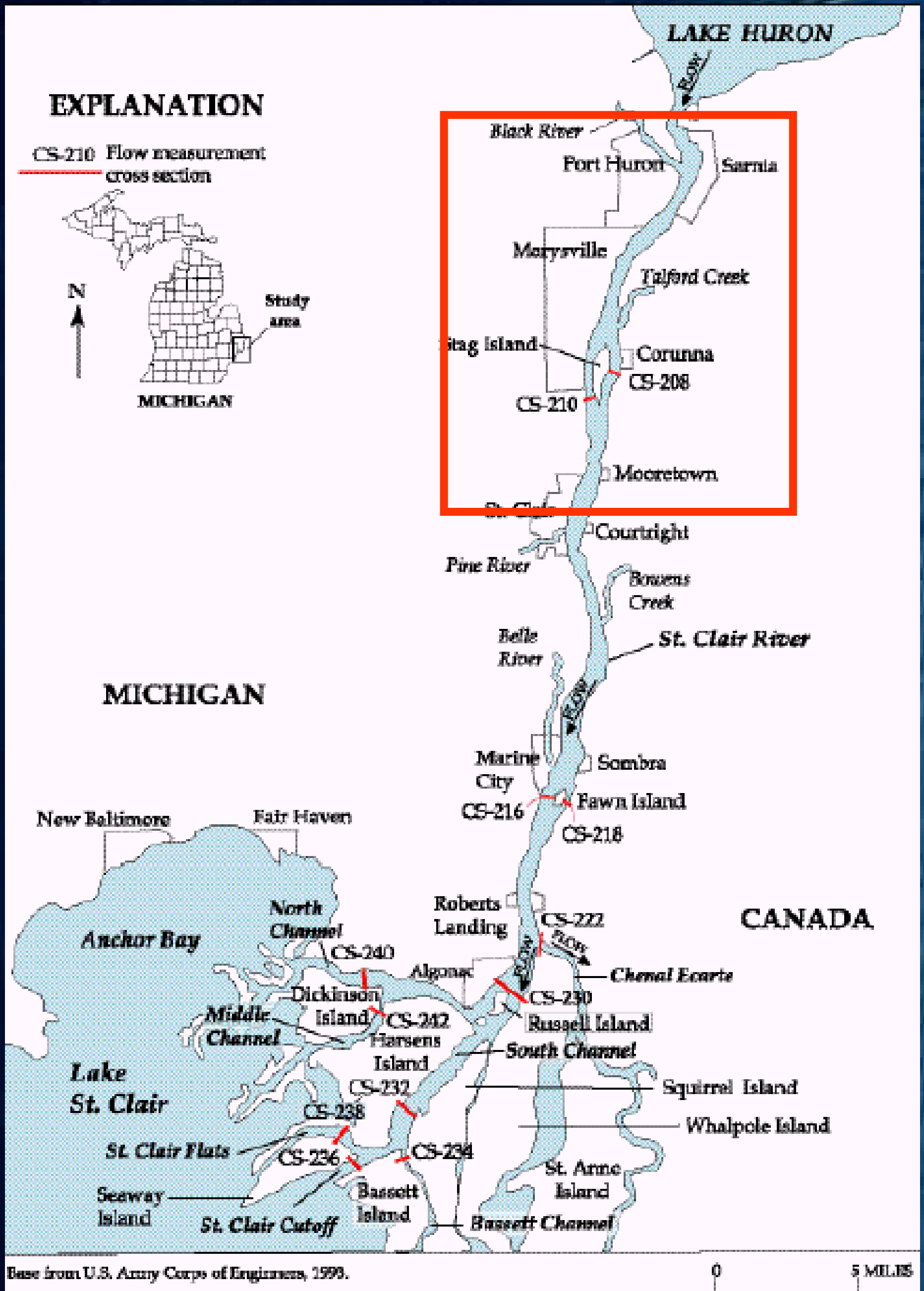








Upper St. Clair River



Example One

Using Numerical Model to Estimate the Volumetric Transport of Water from Lake Huron to Lake St. Clair

To compare the model results with 3D ADCP data

Project Chief:
David J. Holtslag
Michigan District

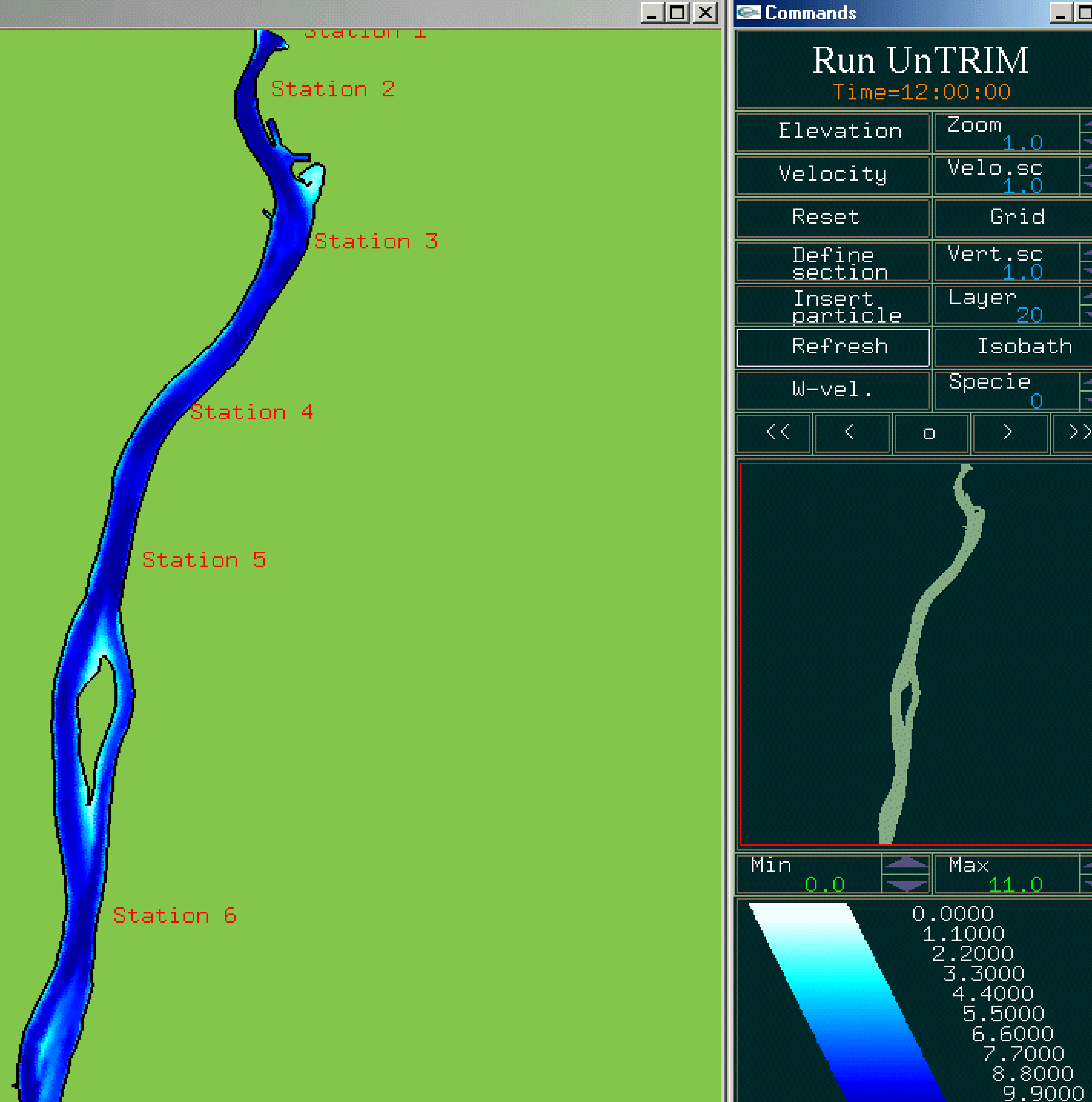
Modeled Area:

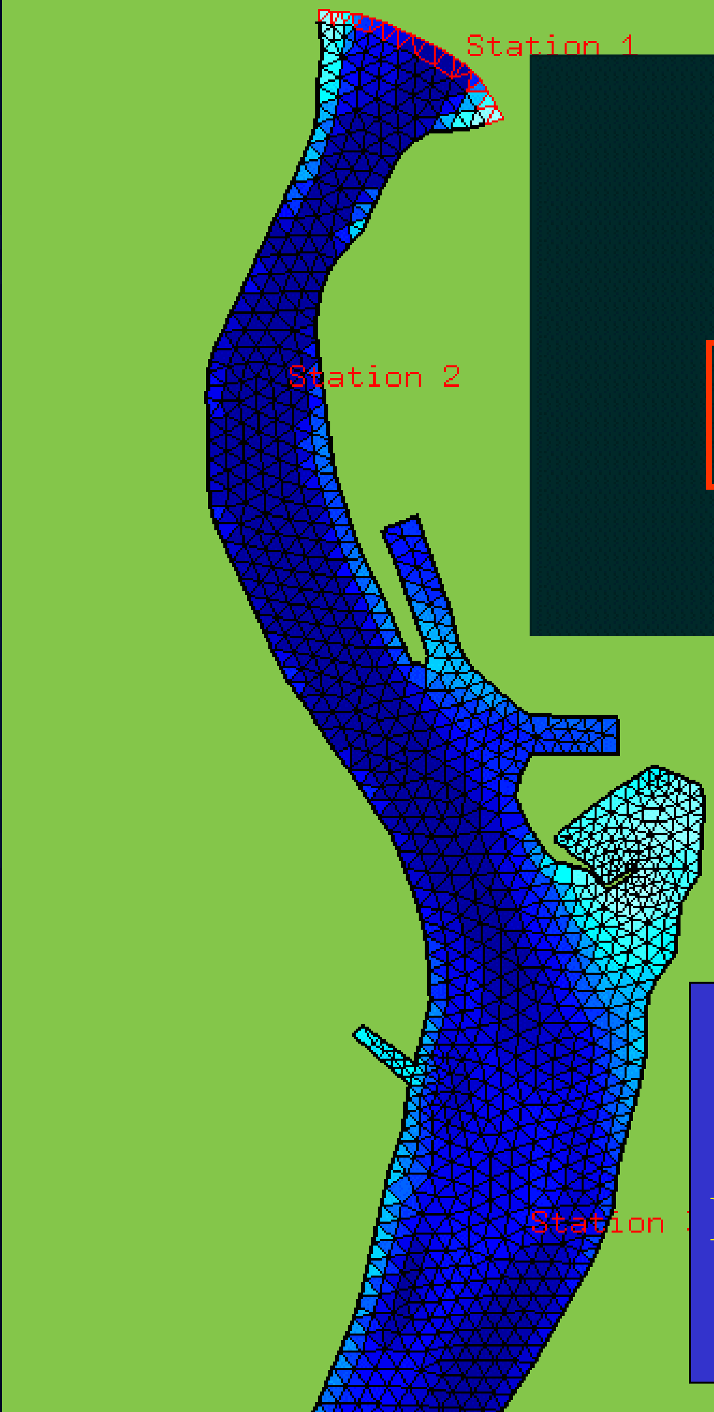
6 km x 22 km

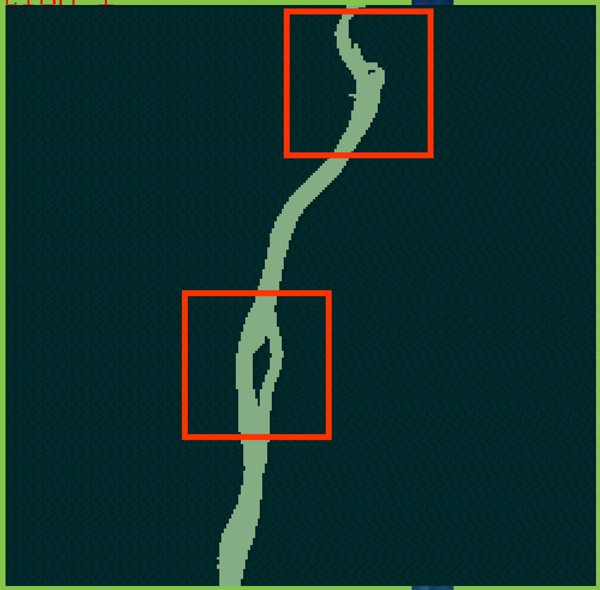
20 layers

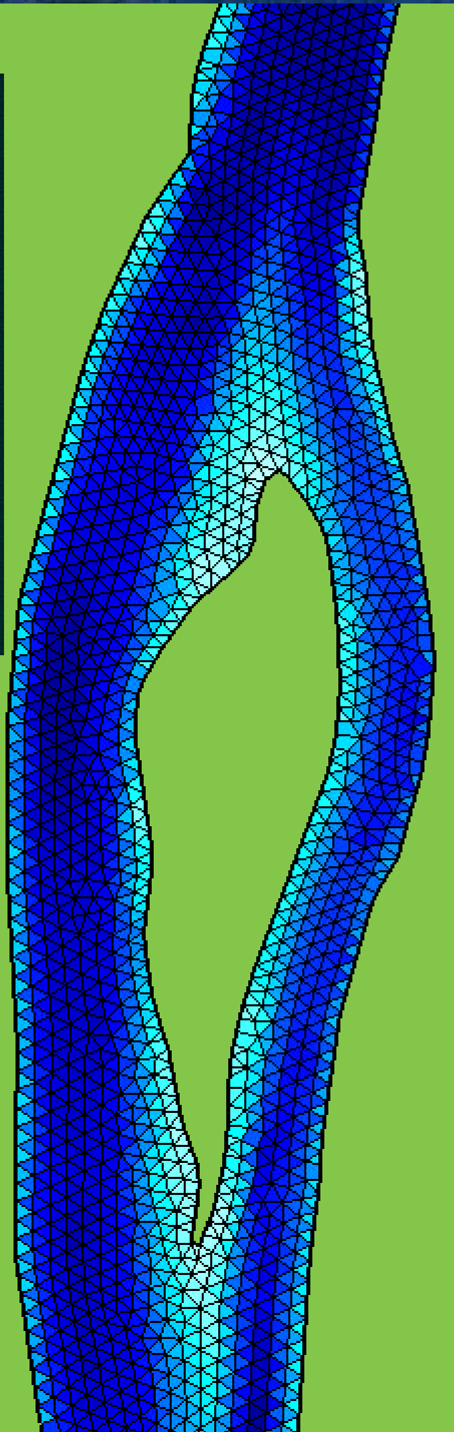
12997 sides

114,462 faces



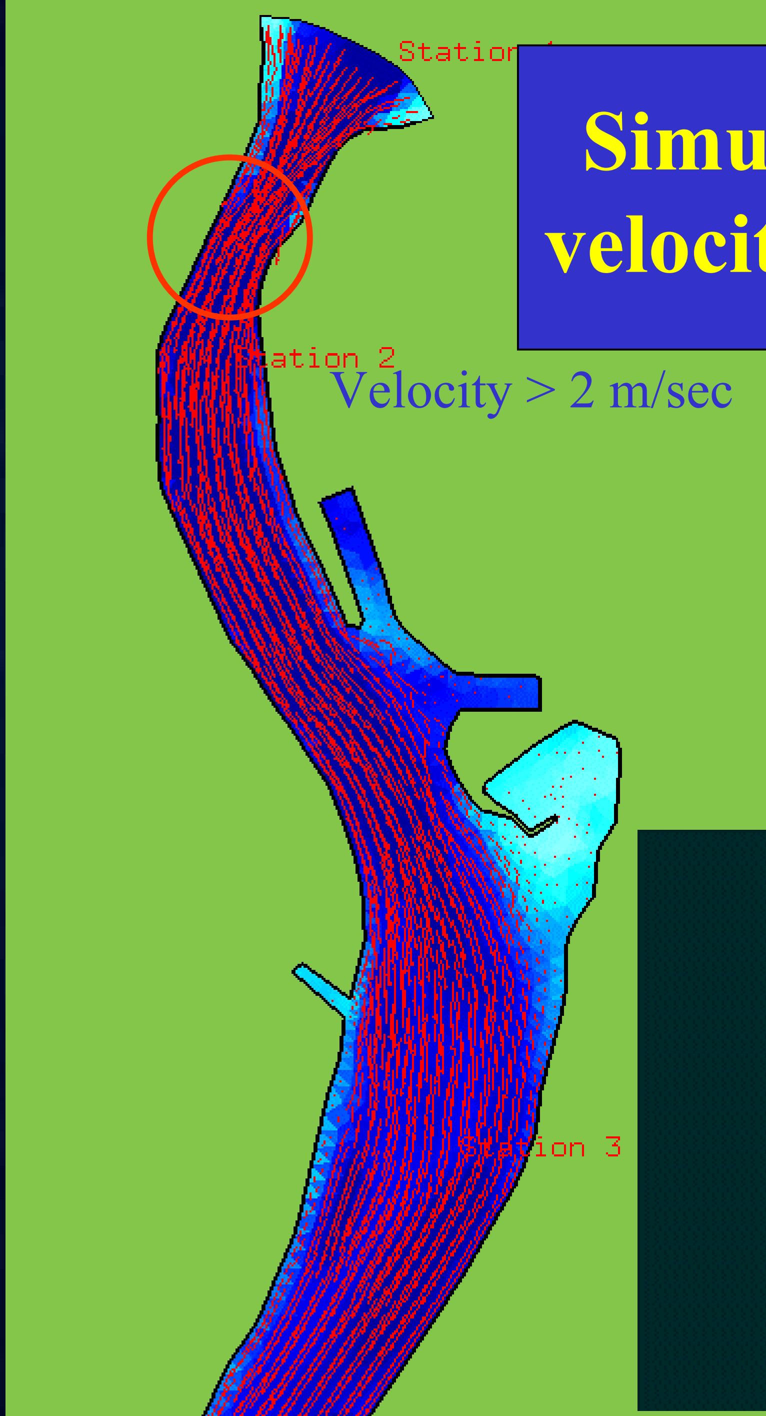


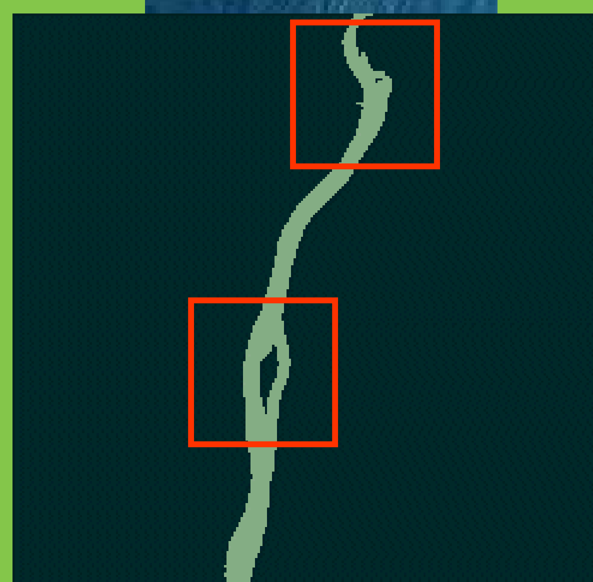


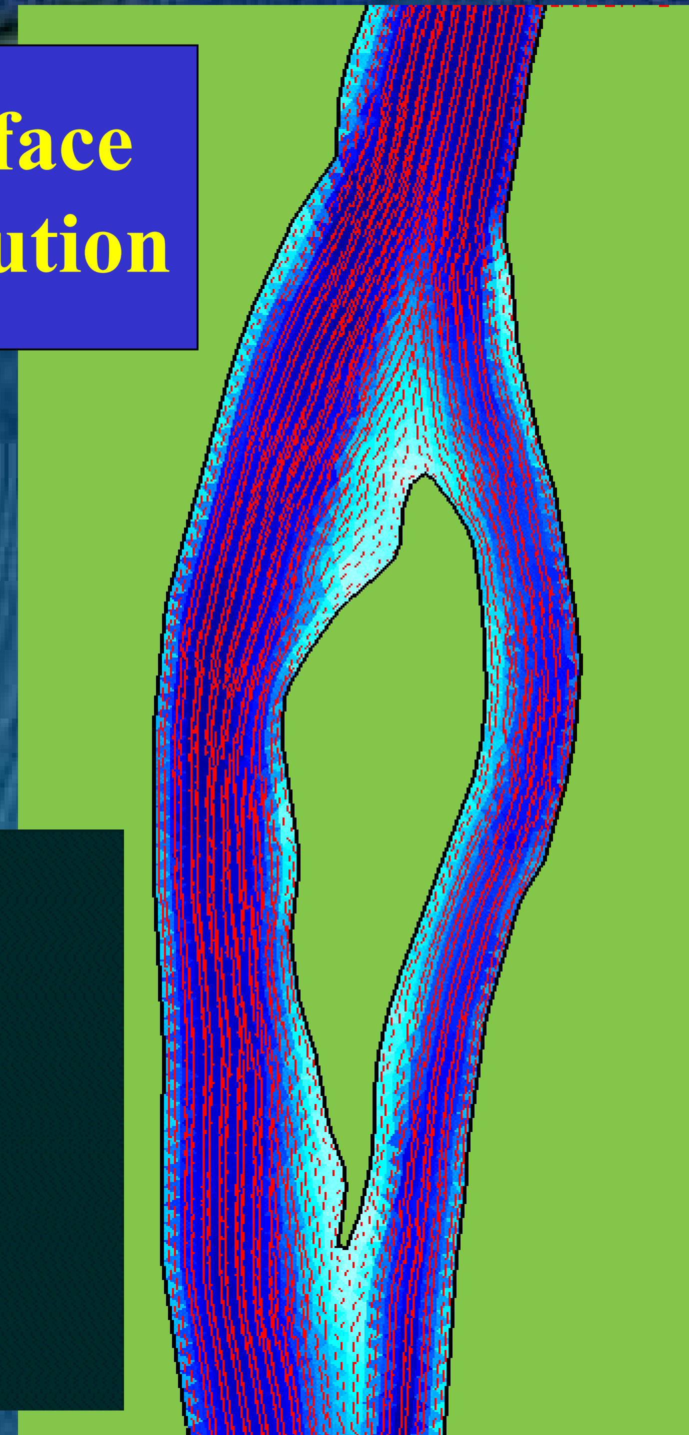


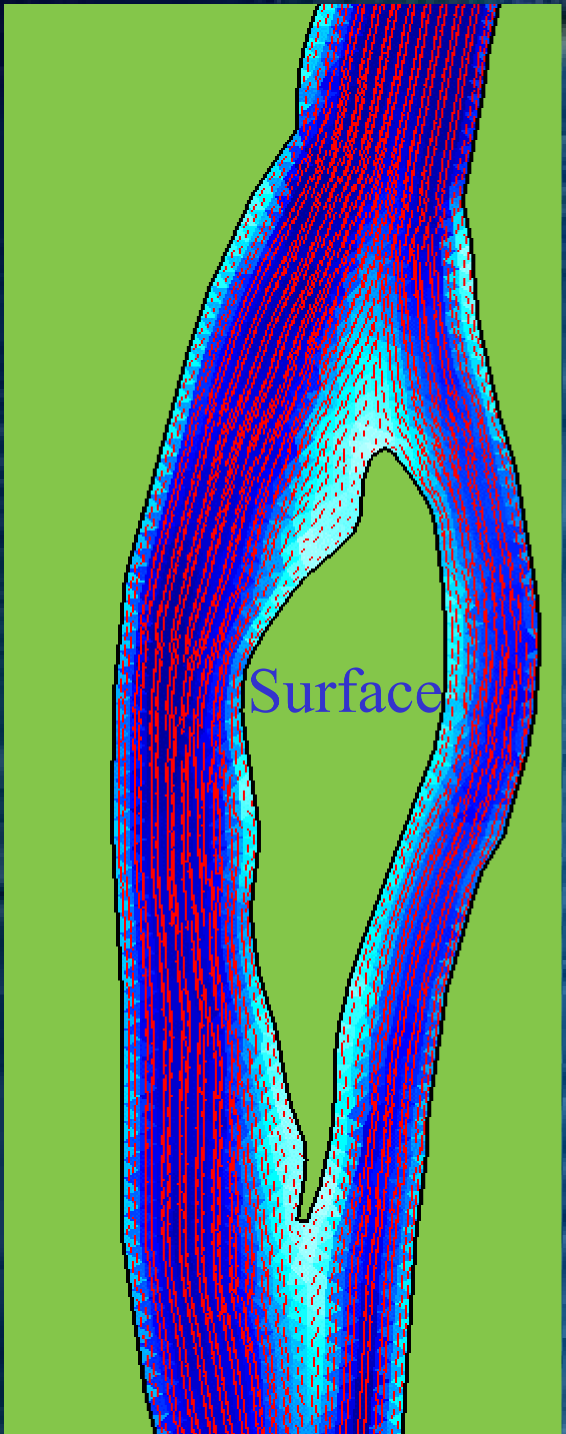
Unstructured Grid Representation of the River

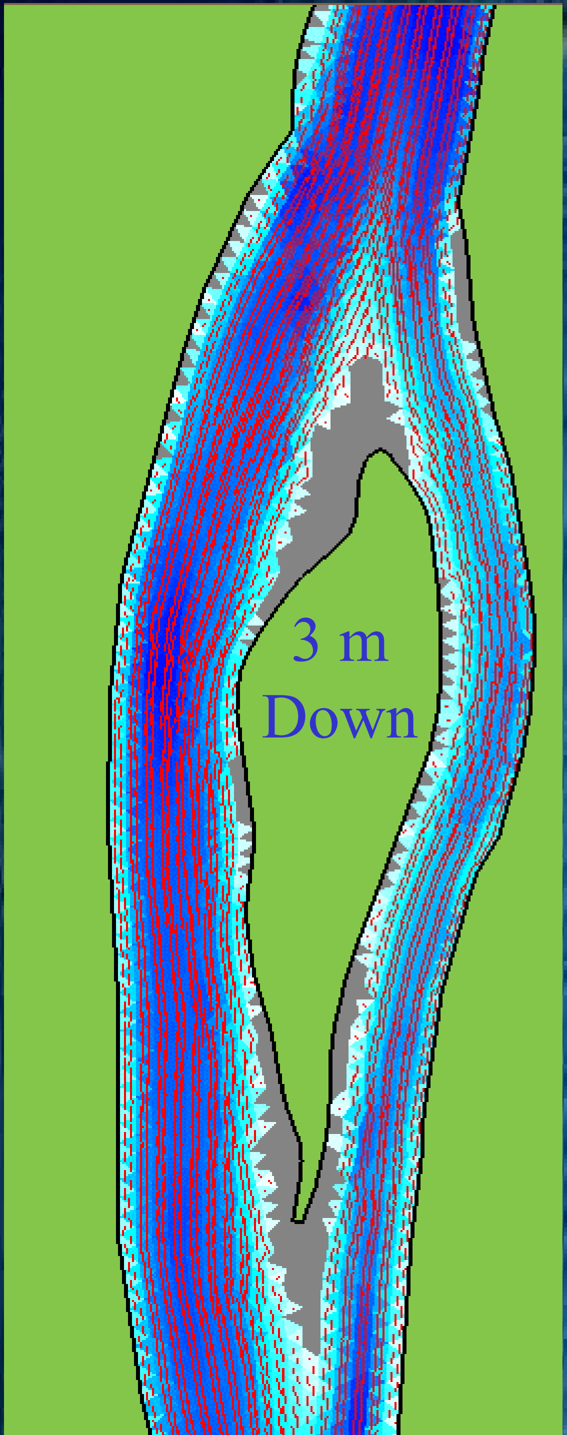
Simulated surface velocity distribution

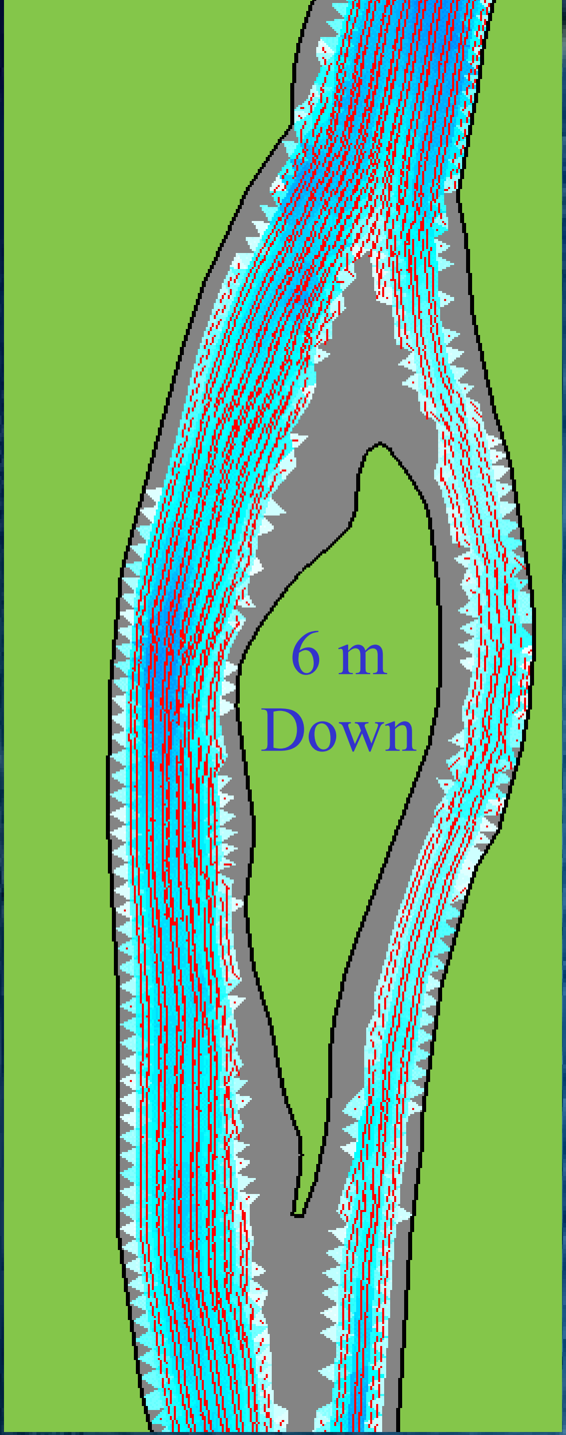


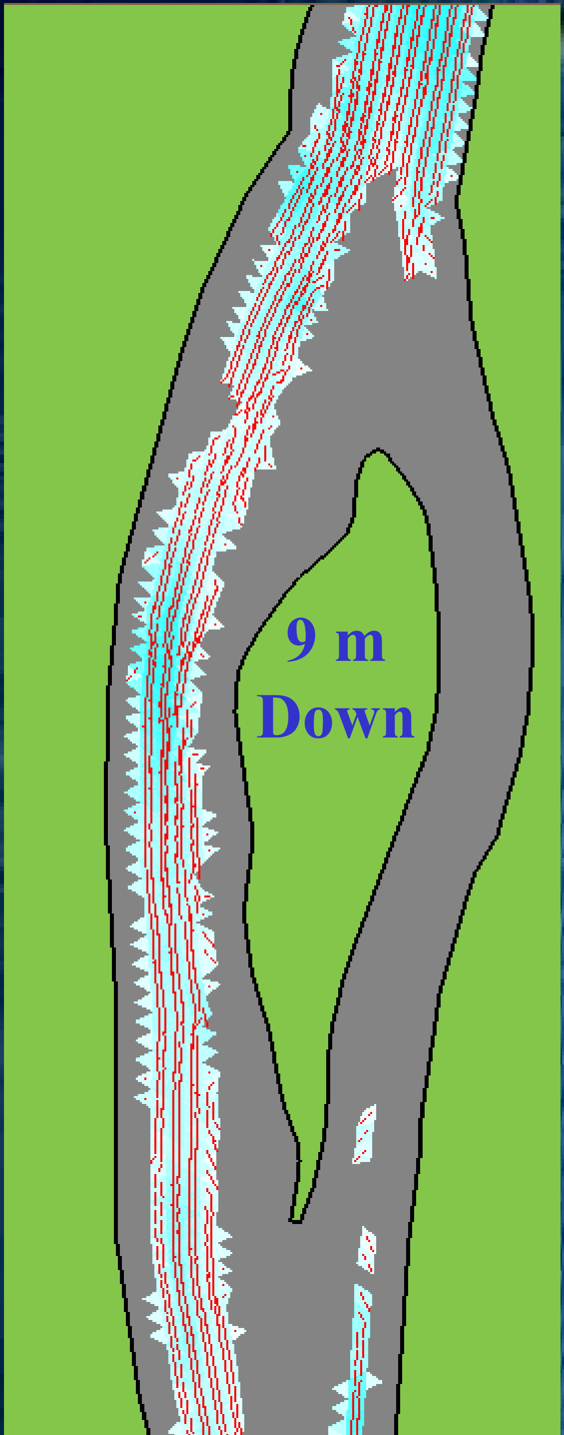





Surface


3 m
Down

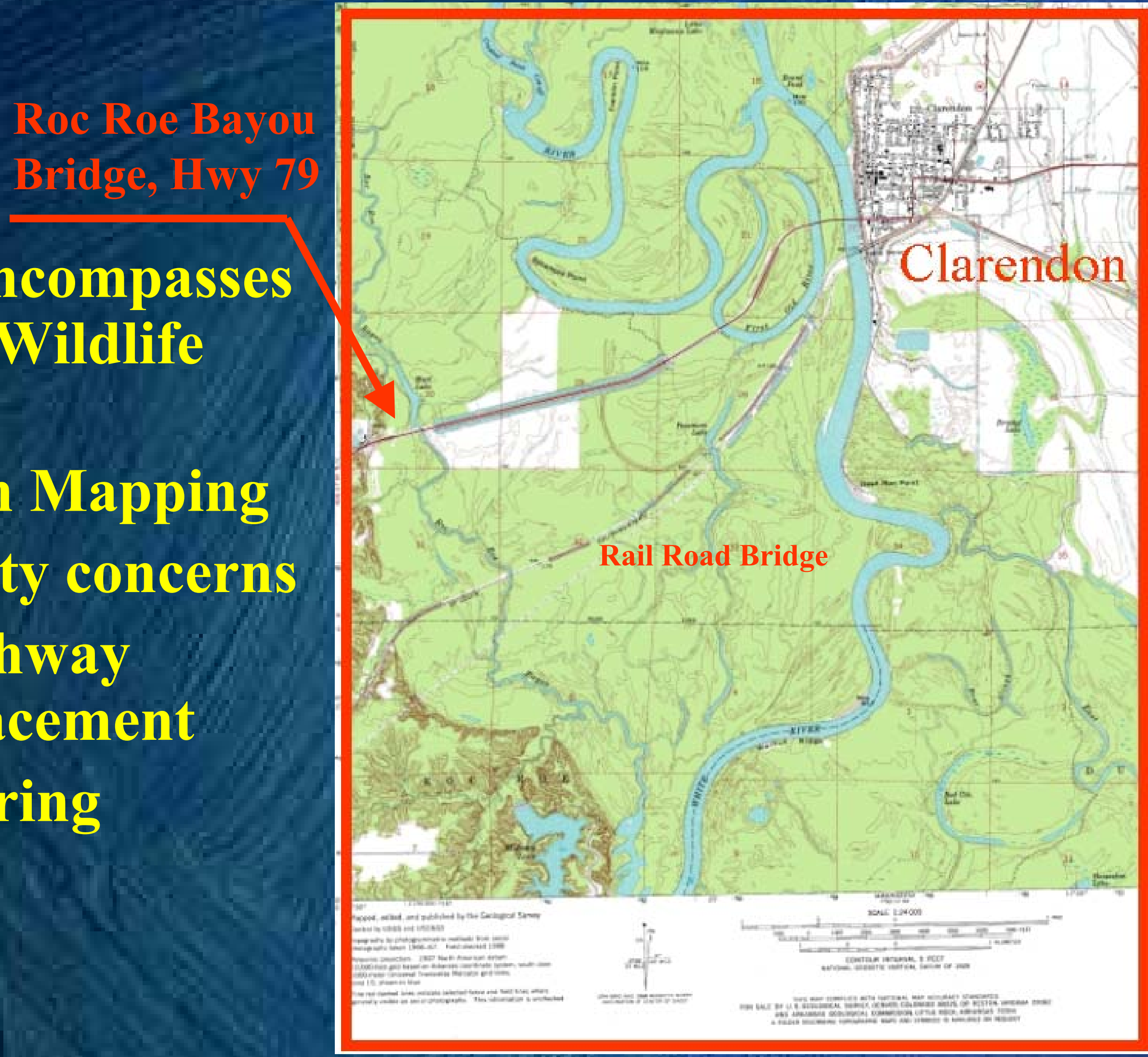

6 m
Down


9 m
Down

Project: Modeling Study of White River, Arkansas

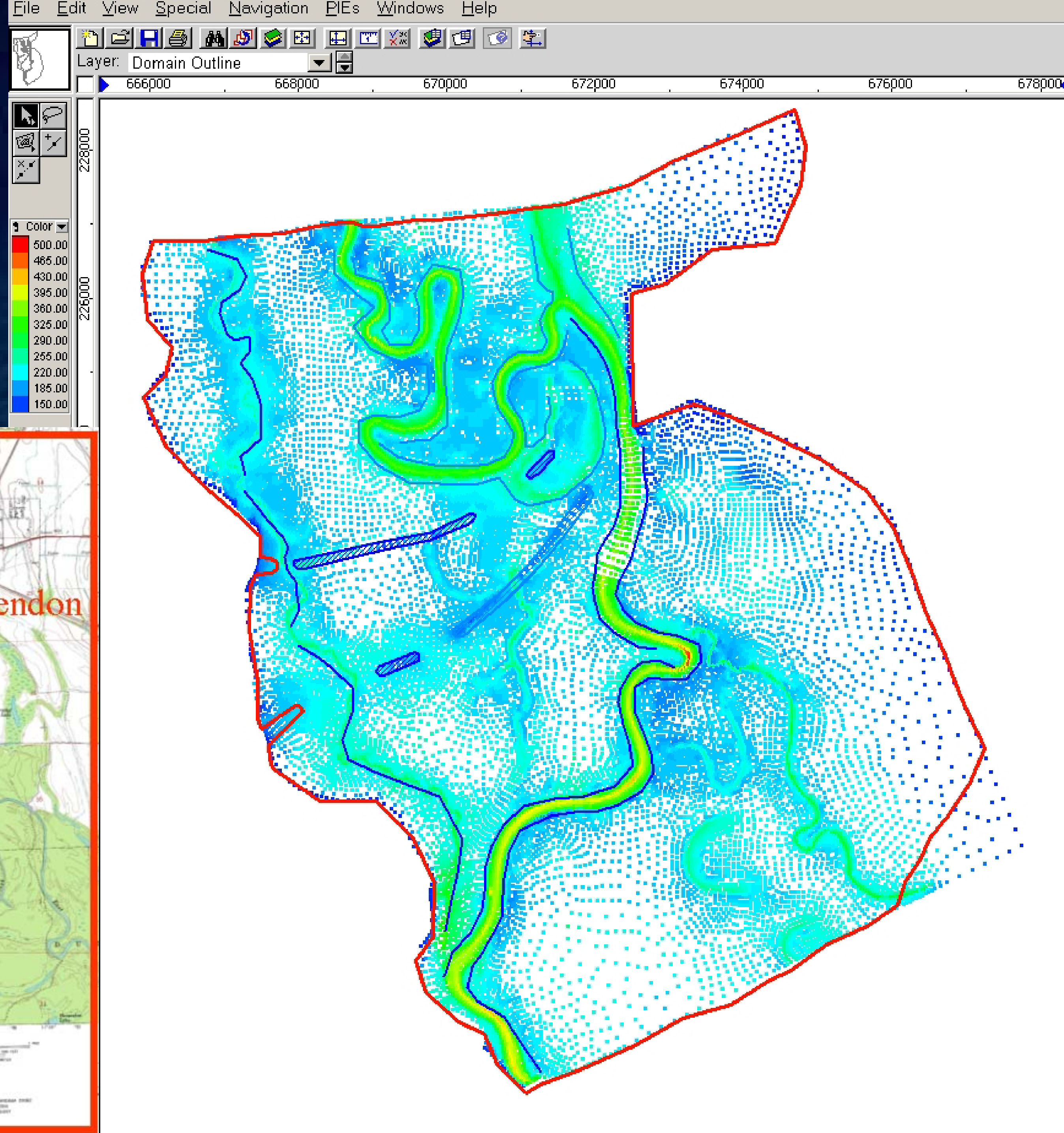
Project: Jaysson Funkhouser and C. Shane Barks

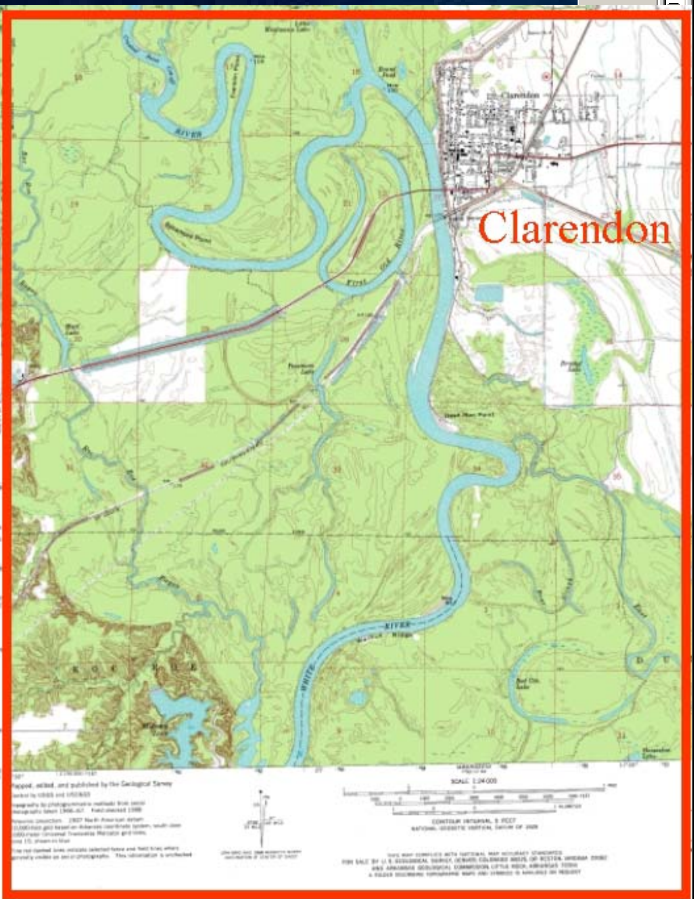
- Study area (34 mi²) Encompasses parts of two National Wildlife Refuges
- Floodplain Inundation Mapping
- Backwater and Velocity concerns
- Provide Inputs to Highway Bridge Design and Placement
- Highway Bridge Scouring



Approximate Area Modeled

Internal Contours to guide and control mesh generation

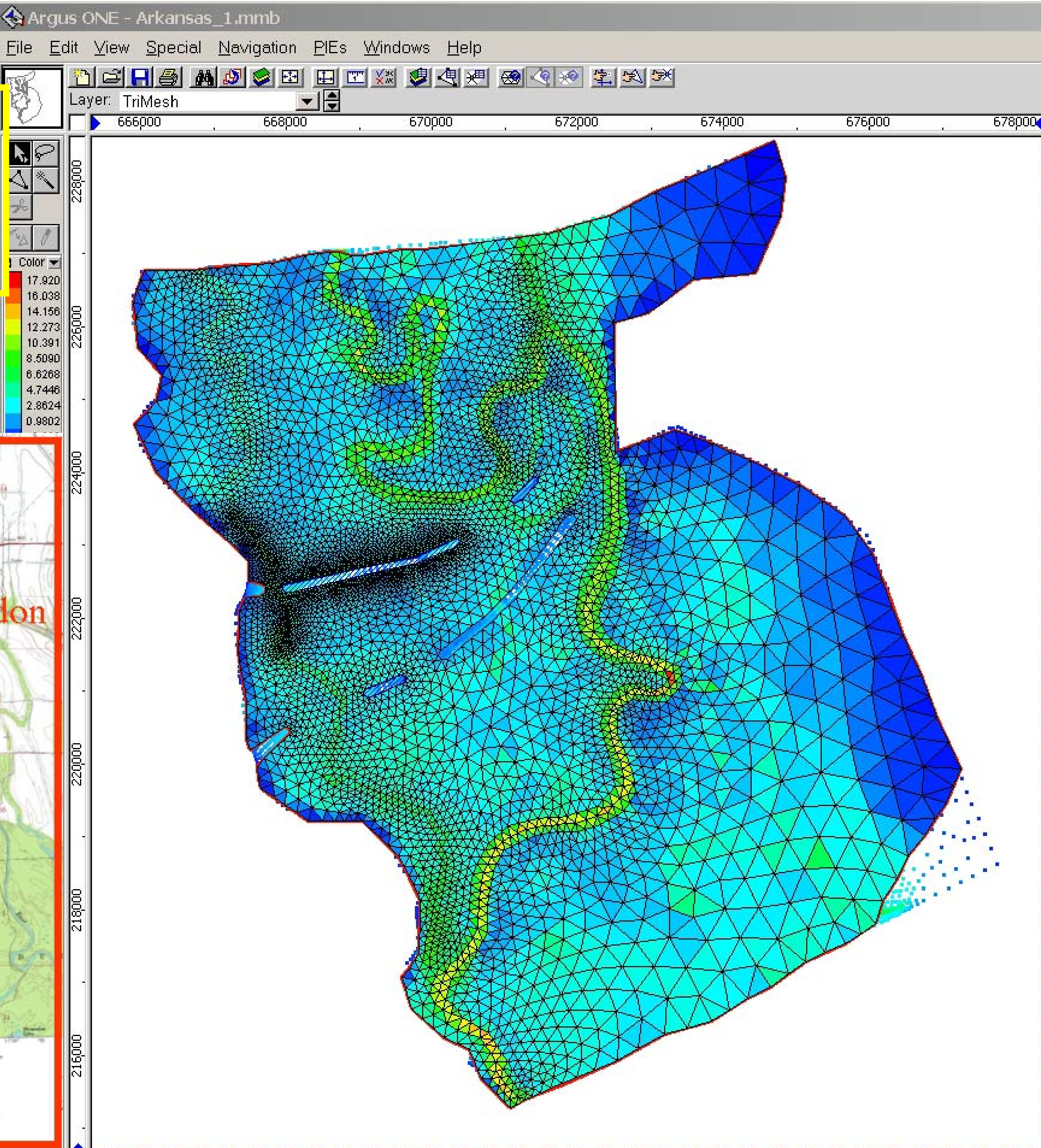


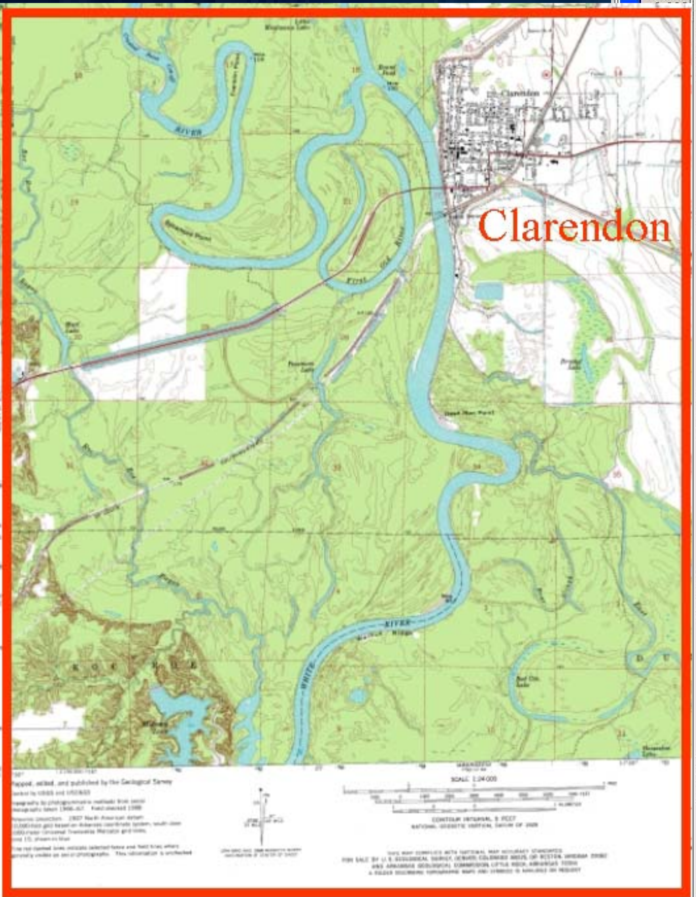


Generated Mesh:

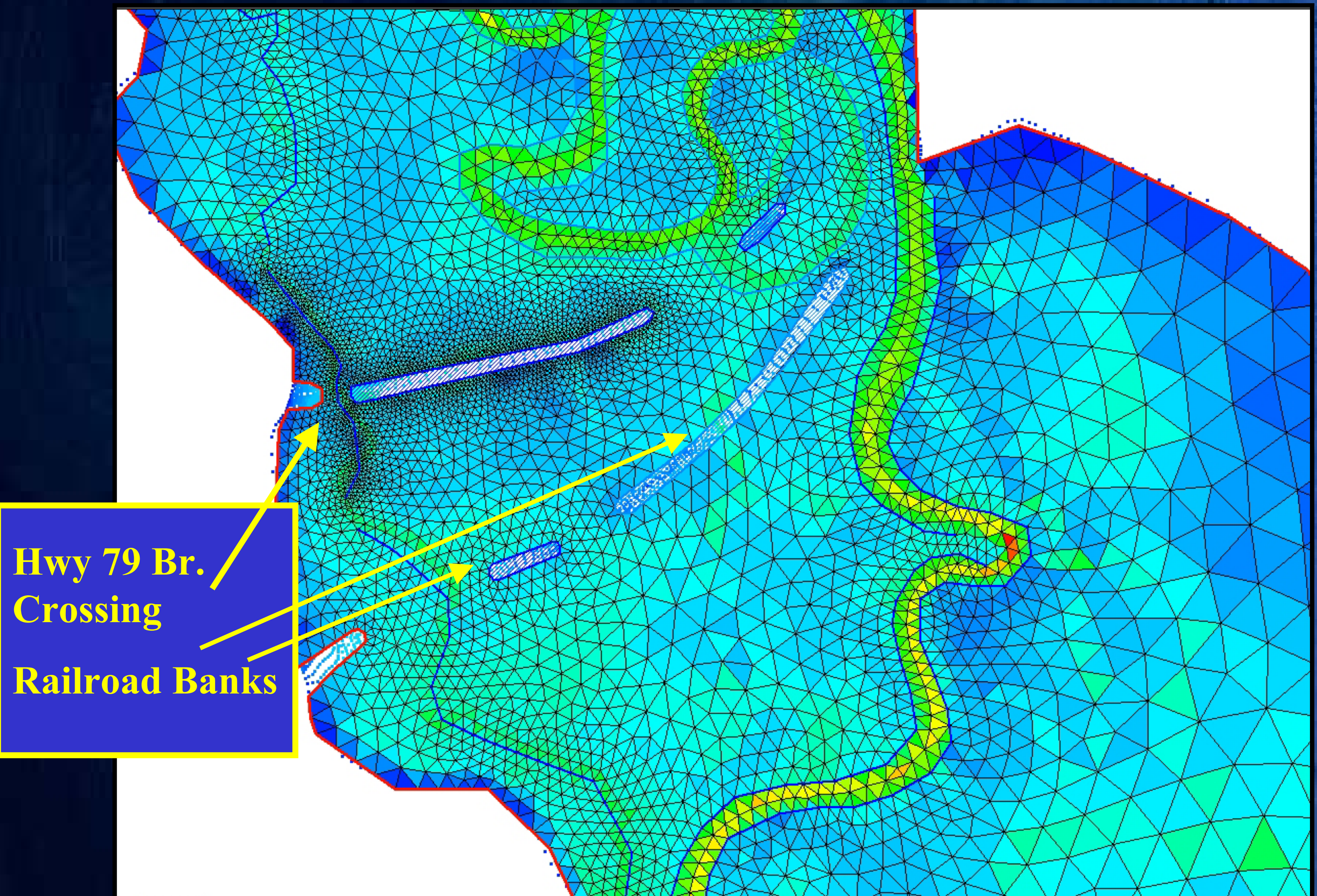
 # Polygons = 10,259

 # Sides = 15,851

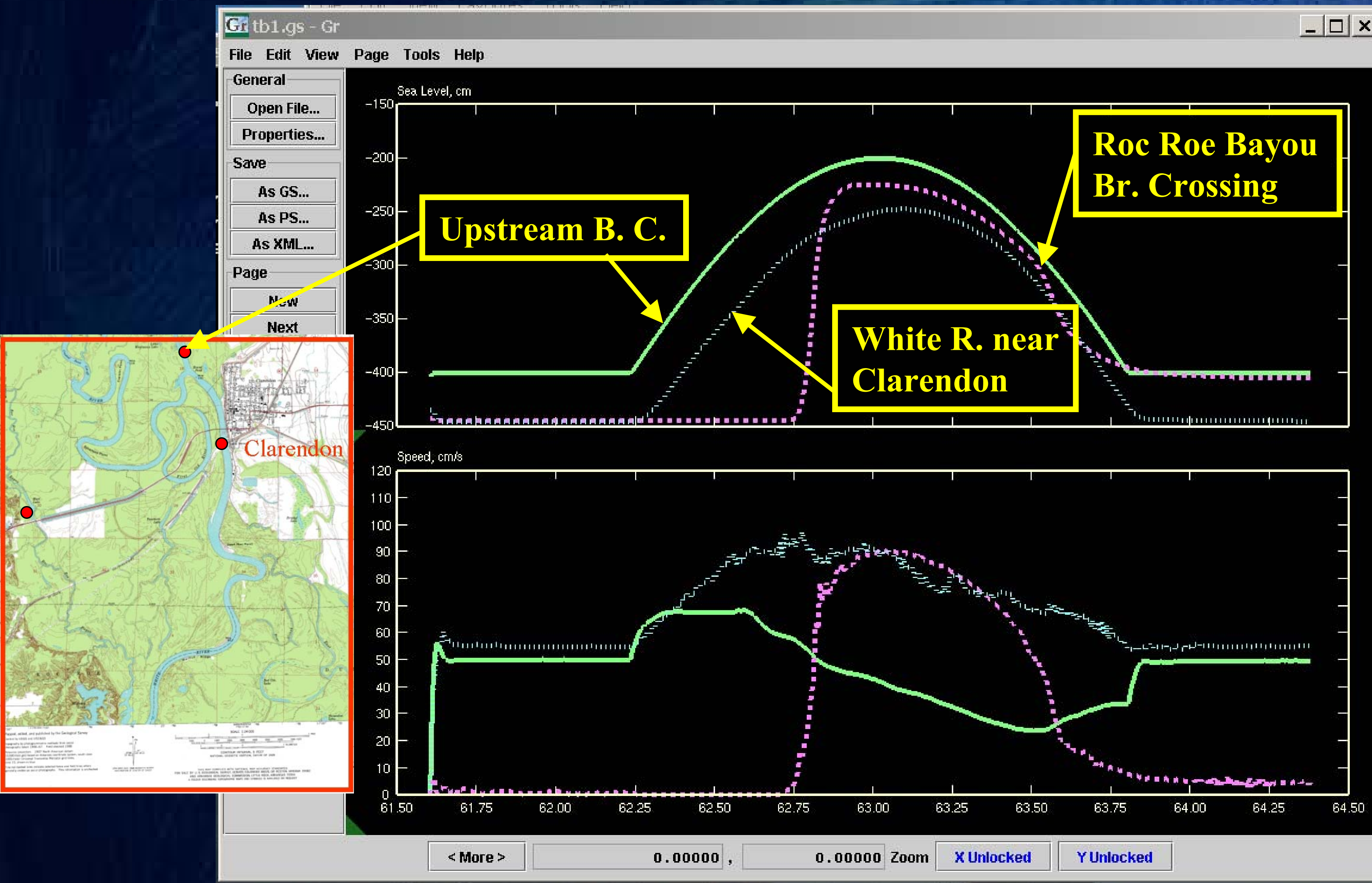


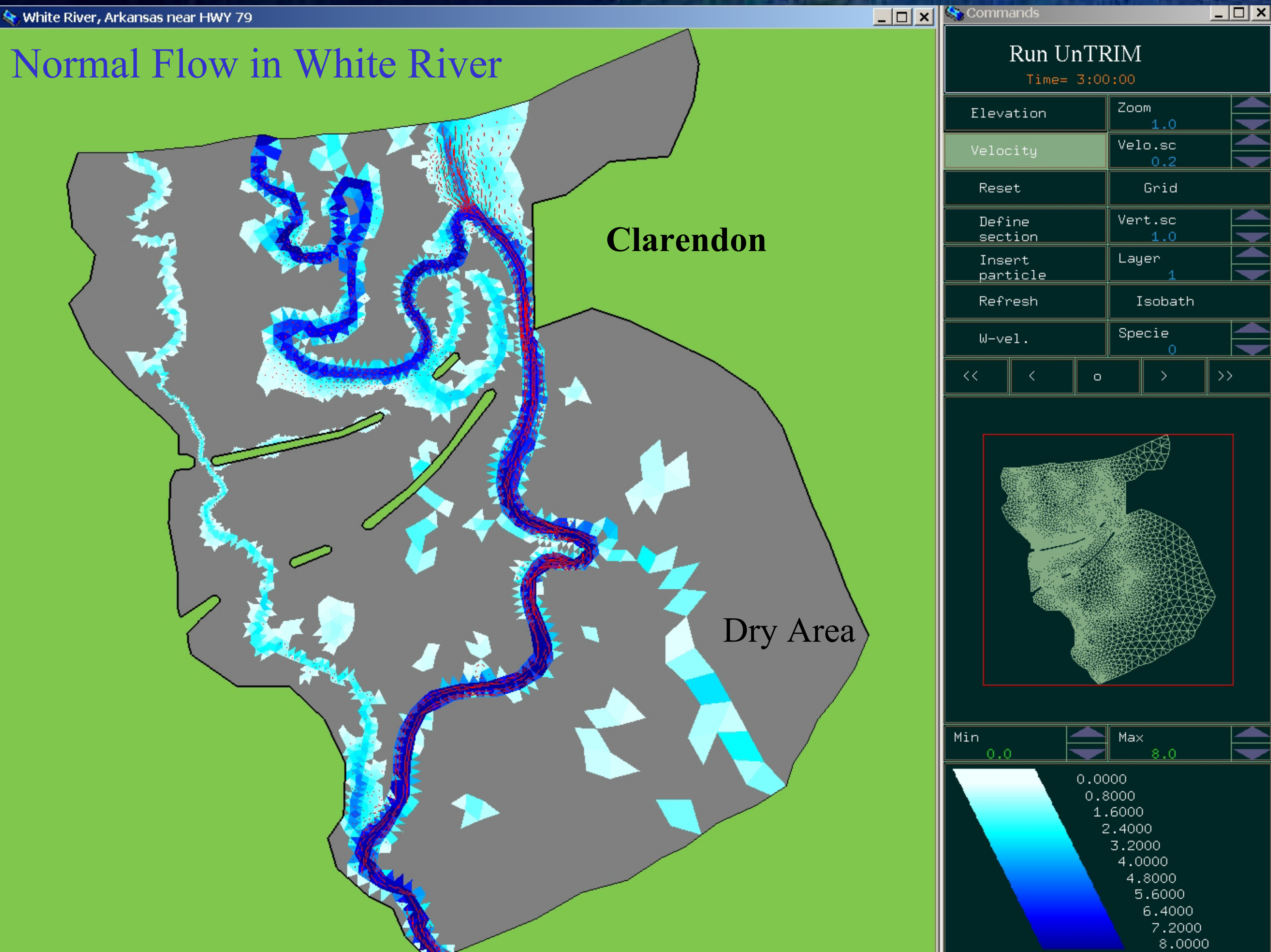


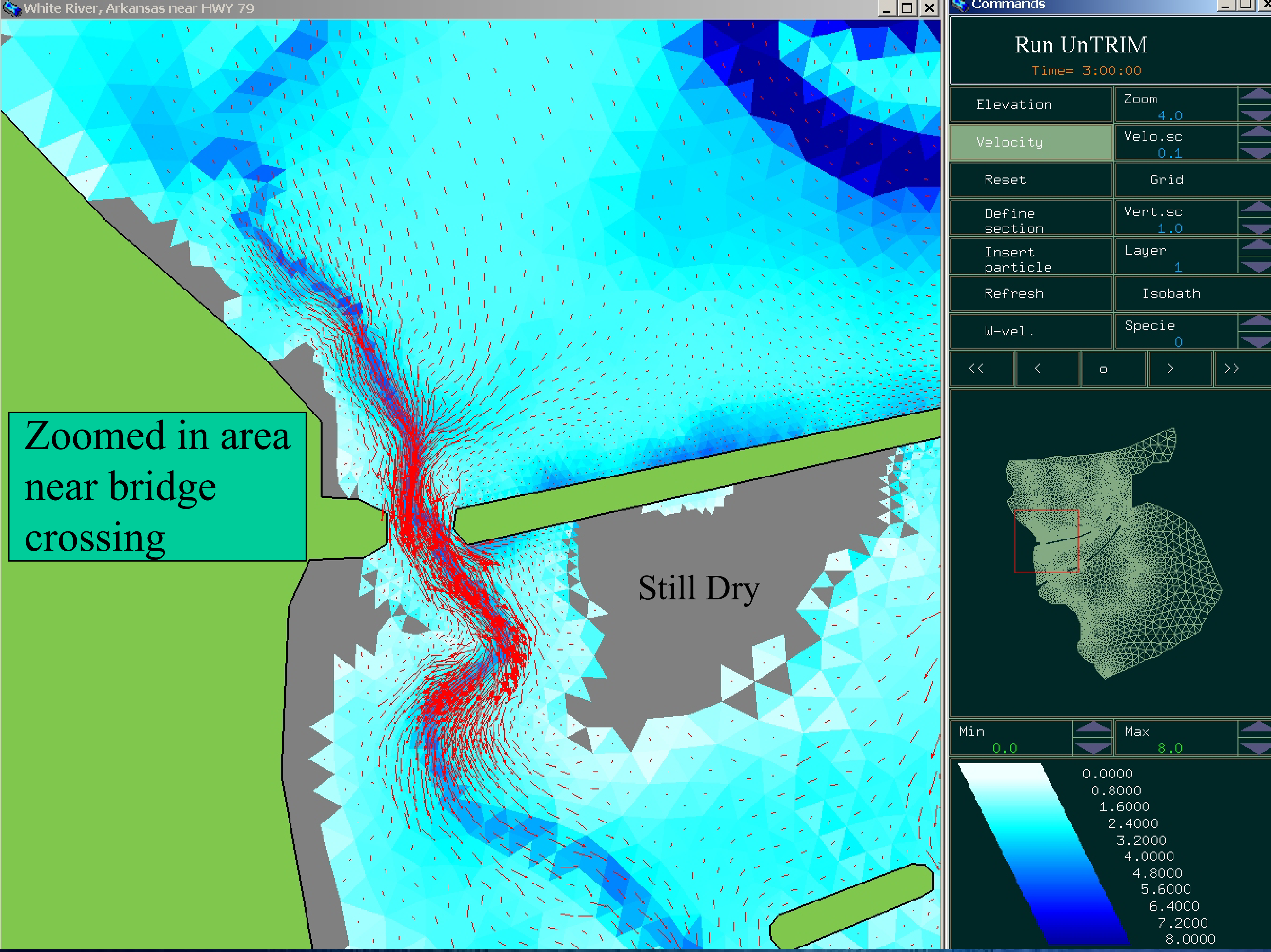
Closer look at mesh guided by internal contours

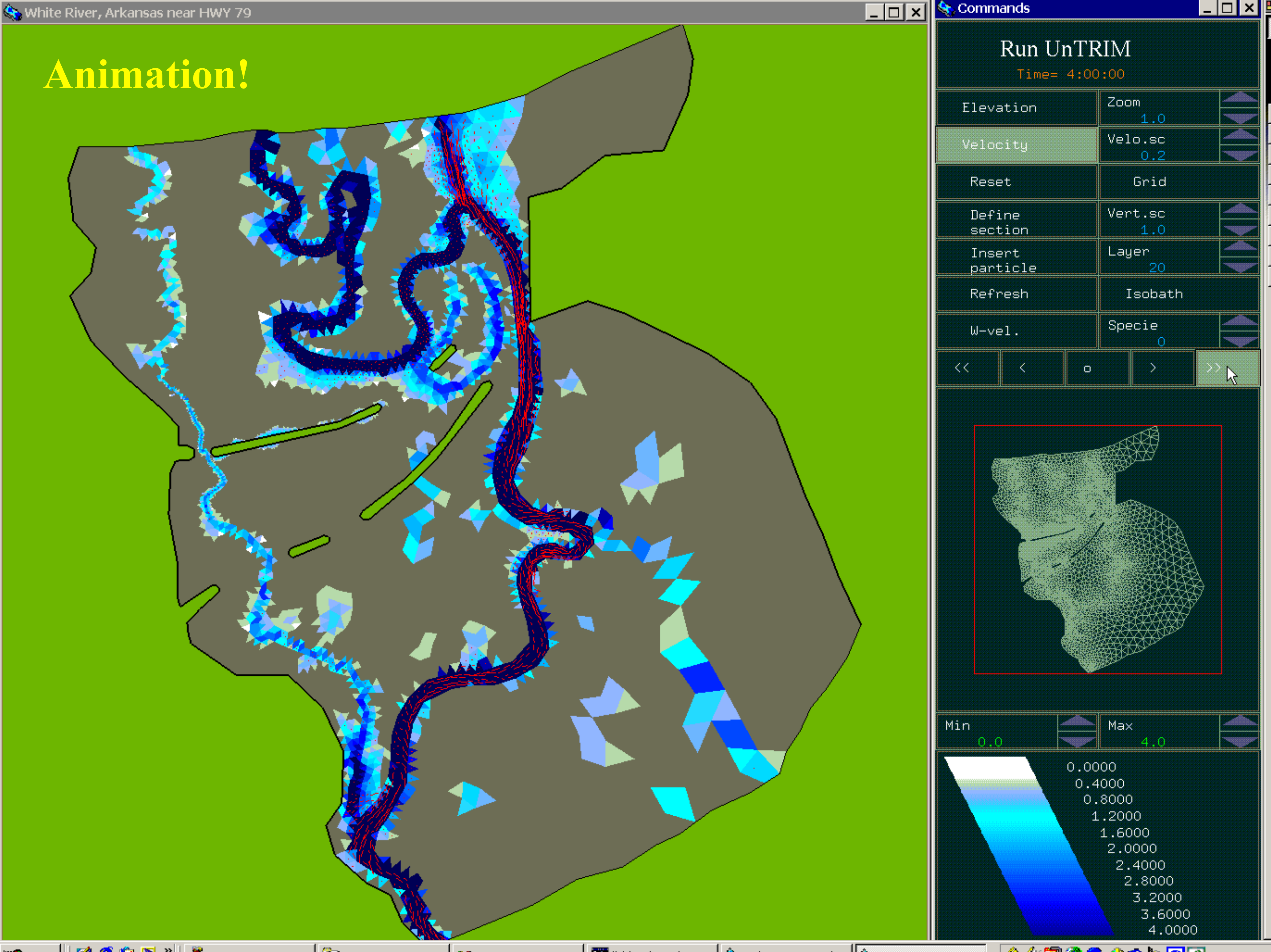


Hydrograph and time-series of a simulated flood




White River, Arkansas near HWY 79
Normal Flow in White River
Clarendon
Dry Area
Commands
Run UnTRIM
Time= 3:00:00
Elevation
Zoom
1.0
Velocity
Velo.sc
0.2
Reset
Grid
Define section
Vert.sc
1.0
Insert particle
Layer
1
Refresh
Isobath
W-vel.
Specie
0
<<
<
o
>
>>
Min
0.0
Max
8.0
0.0000
0.8000
1.6000
2.4000
3.2000
4.0000
4.8000
5.6000
6.4000
7.2000
8.0000


White River, Arkansas near HWY 79
Zoomed in area near bridge crossing
Still Dry
Commands
Run UnTRIM
Time= 3:00:00
Elevation
Zoom
4.0
Velocity
Velo.sc
0.1
Reset
Grid
Define section
Vert.sc
1.0
Insert particle
Layer
1
Refresh
Isobath
W-vel.
Specie
0
<<
<
o
>
>>
Min
0.0
Max
8.0
0.0000
0.8000
1.6000
2.4000
3.2000
4.0000
4.8000
5.6000
6.4000
7.2000
8.0000


White River, Arkansas near HWY 79
Animation!
Commands
Run UnTRIM
Time= 4:00:00
Elevation
Zoom
1.0
Velocity
Velo.sc
0.2
Reset
Grid
Define section
Vert.sc
1.0
Insert particle
Layer
20
Refresh
Isobath
W-vel.
Specie
0
<<
<
o
>
>>
Min
0.0
Max
4.0
0.0000
0.4000
0.8000
1.2000
1.6000
2.0000
2.4000
2.8000
3.2000
3.6000
4.0000

An UnTRIM application
Modeling Wind-Driven Circulation in Upper Klamath Lake

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Tamara Wood**

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**U. S. Geological Survey, Portland, OR

I. Background and Issues

II. ADCP Deployment and Results

III. Time-series of Wind Observations

IV. Wind-Driven Circulation

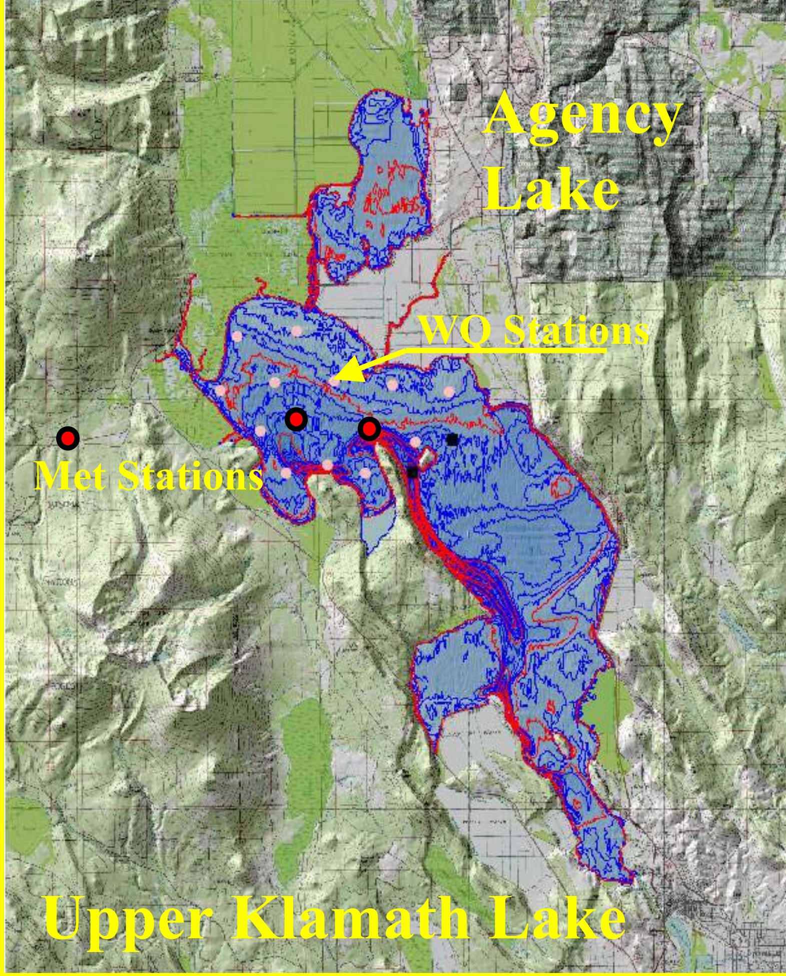
V. Reproducing ADCP Observations

VI. Analyze This and Analyze That

VII. Conclusion (Physics Rules!)

Background

- 120 mi^2 surface area
- Mean depth 8 ft
- Drains 3800 mi^2 phosphorus-rich volcanic soils
- Naturally eutrophic
- Wetlands drained and logging since
- 1900
- Presently hyper-eutrophic
- chlorophyll ~100-300 mg/L
- pH exceeding 9.5
- Ammonia ~ 500 mg/L
- DO approaching zero during a crash
- Massive annual cyanobacteria blooms

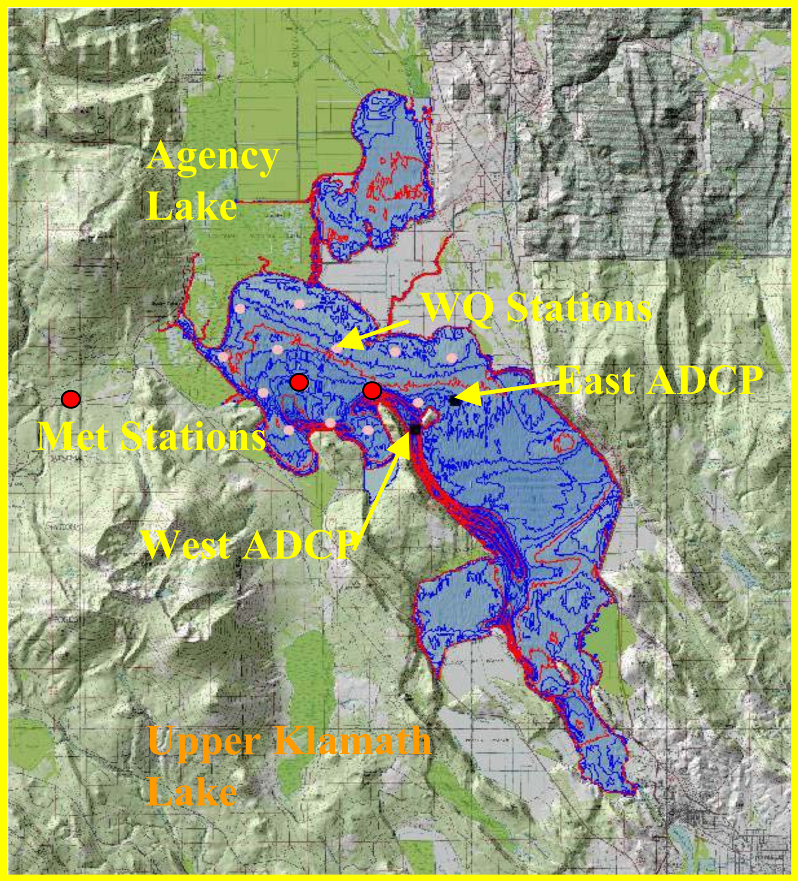


2002: USGS/BOR began a cooperative study of fish behavior in response to lake water quality conditions

Two main tasks:

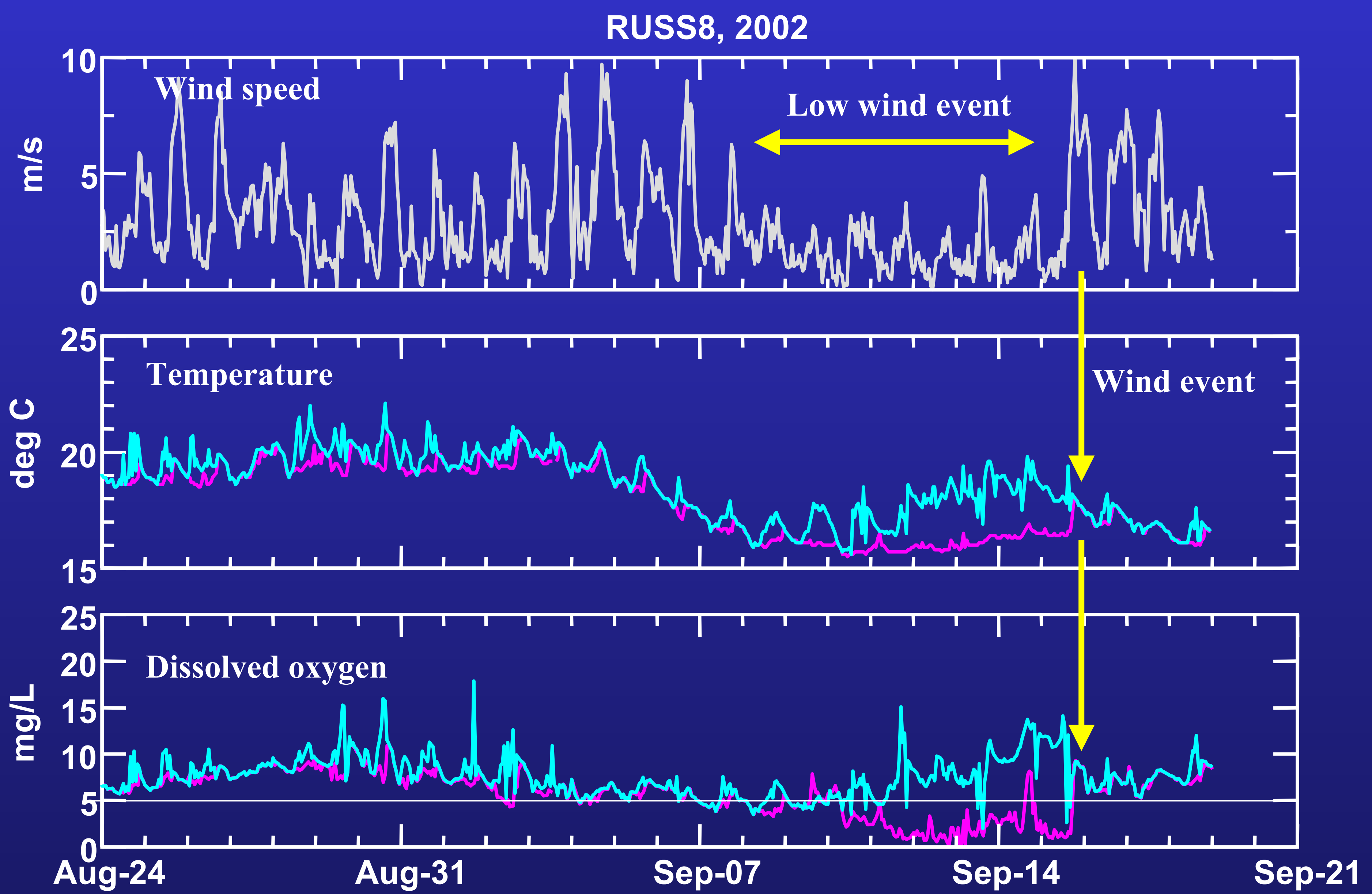
- **Tracking of tagged adult suckers using telemetry over northern 1/3 of UKL**
- **A network of continuous WQ monitors that would provide a “map” of water quality**

Wind Observations on the lake (Russ7)



Issues with wind time-series:

1. **Magnetic north**
2. **Data gaps or irregular time intervals**



RUSS8, 2002
Wind speed
Low wind event
m/s
10
5
0
Temperature
Wind event
deg C
25
20
15
Dissolved oxygen
mg/L
25
20
15
10
5
0
Aug-24
Aug-31
Sep-07
Sep-14
Sep-21

Sequence of Events Leading to a Fish Kill (prevailing wisdom):

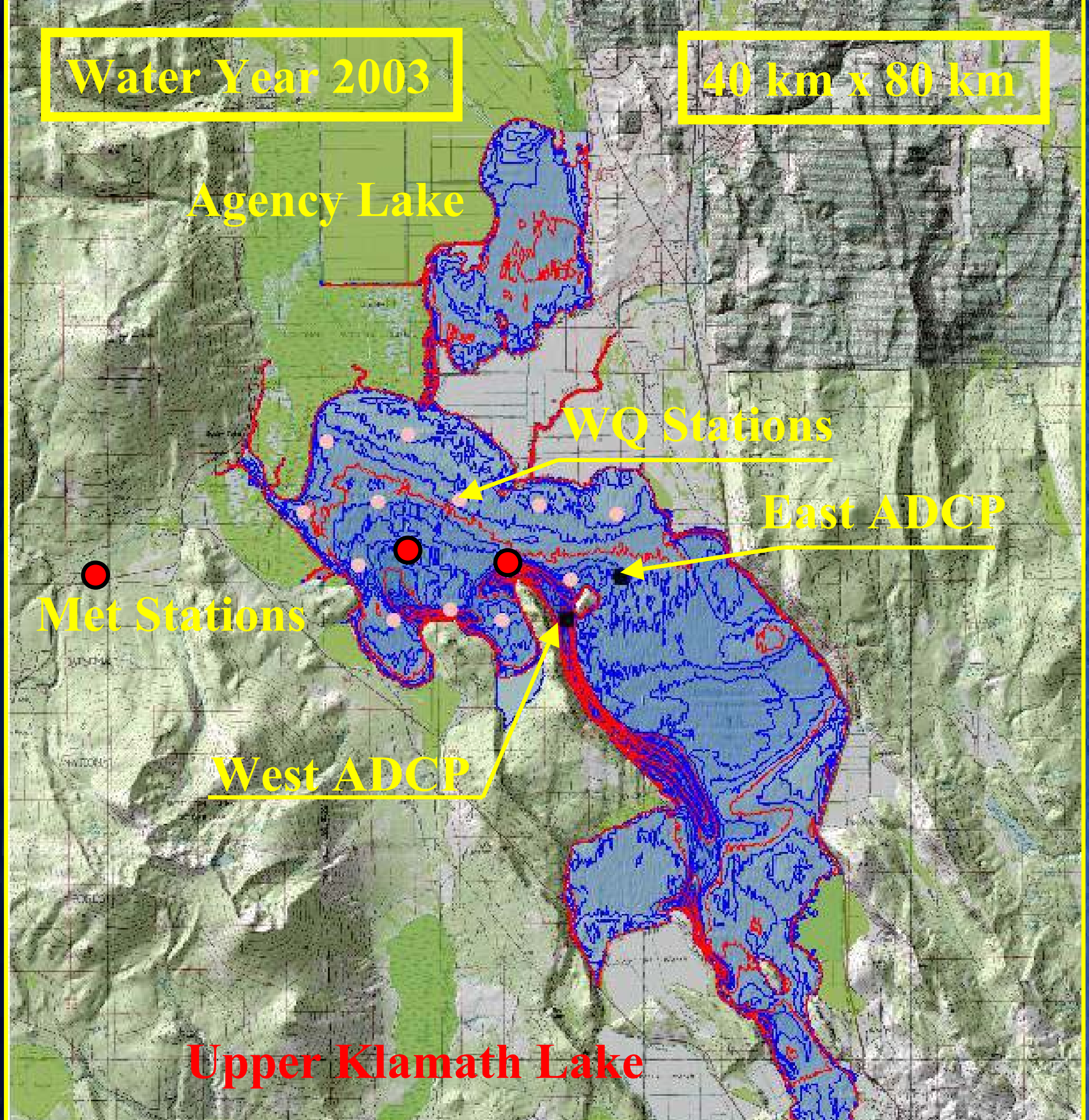
Extended period of thermal stability

Limited mixing in the water column

Very low near-bottom dissolved oxygen

Extended Low DO in water column

Fish Kill


Water Year 2003
40 km x 80 km
Agency Lake
WQ Stations
East ADCP
Met Stations
West ADCP
Upper Klamath Lake

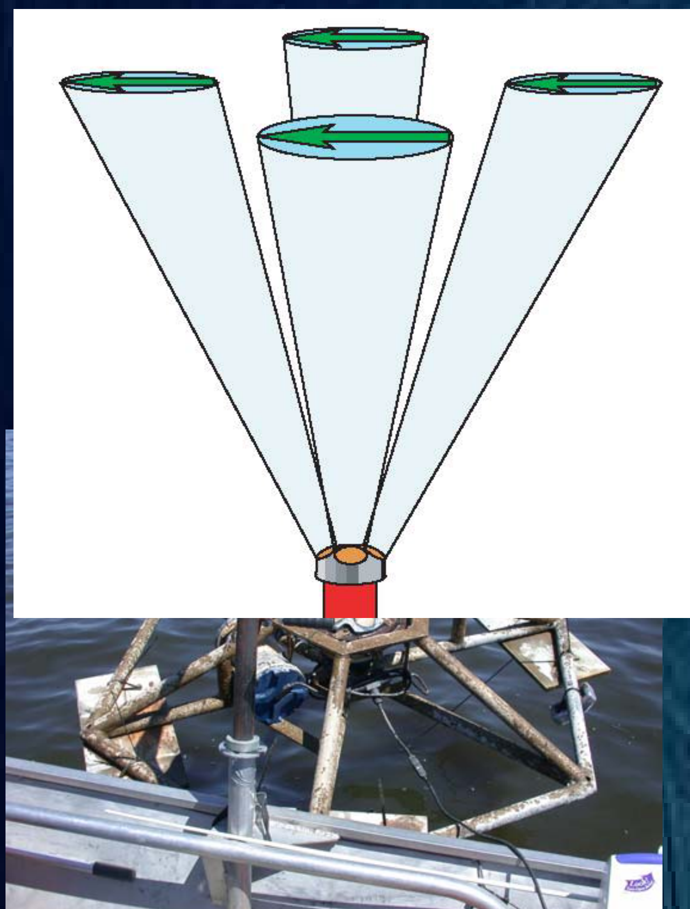
West ADCP Station:

Water depth ~ 8 m

Bin size = 0.2 m

Sampling rate = 30.0 min

Total bins = 34



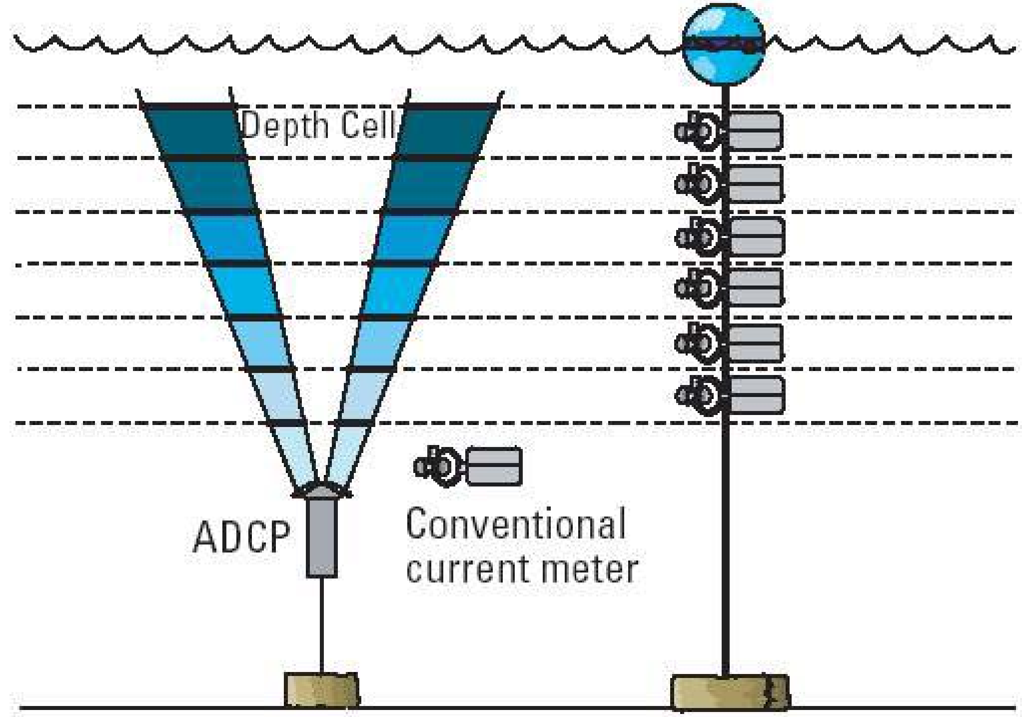


Figure 1.15. Analogy of a conventional current-meter string to an acoustic Doppler current profiler (ADCP) profile.

East ADCP Station:

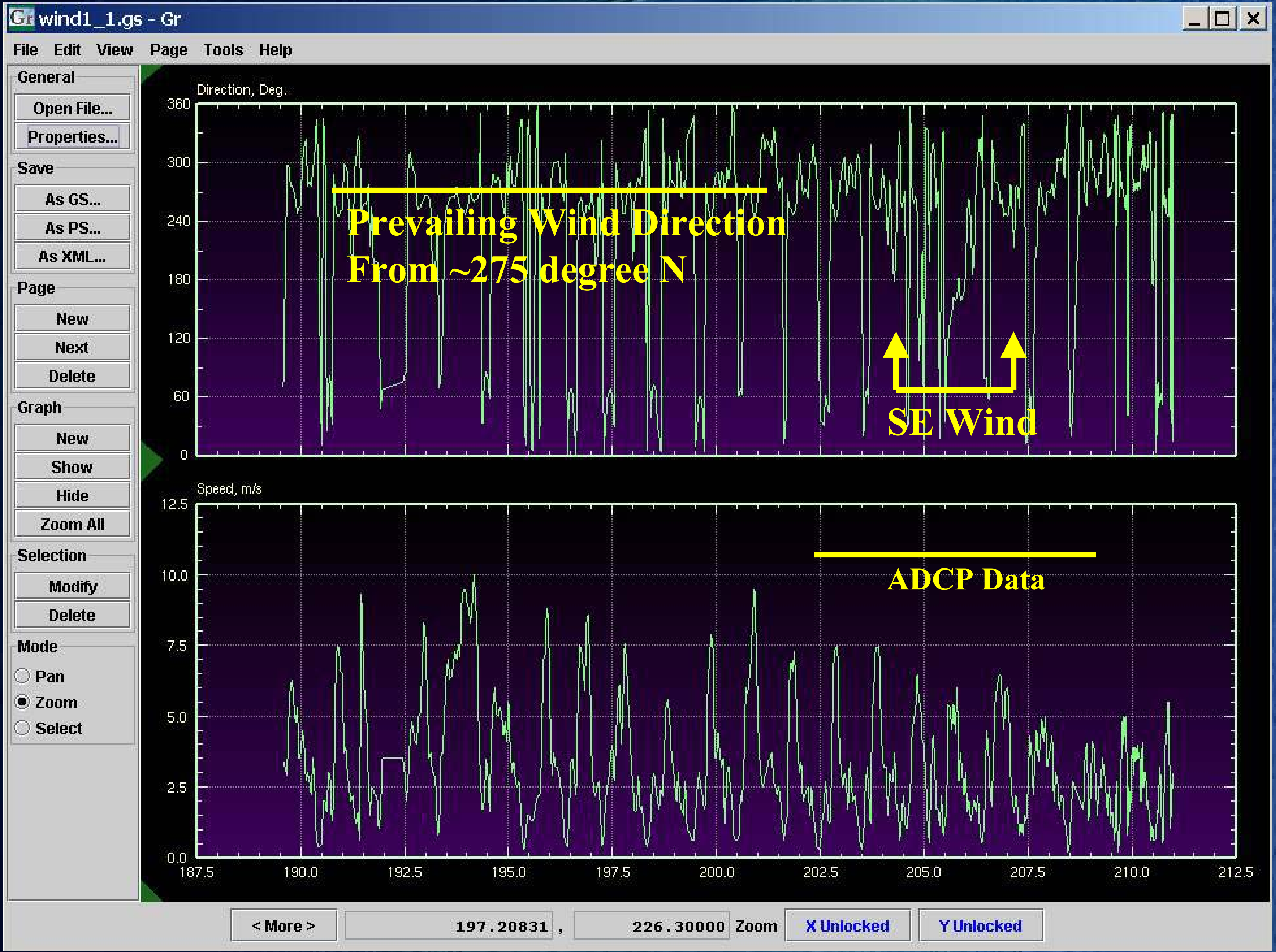
Water depth ~ 3.5 m

Bin size = 0.2 m

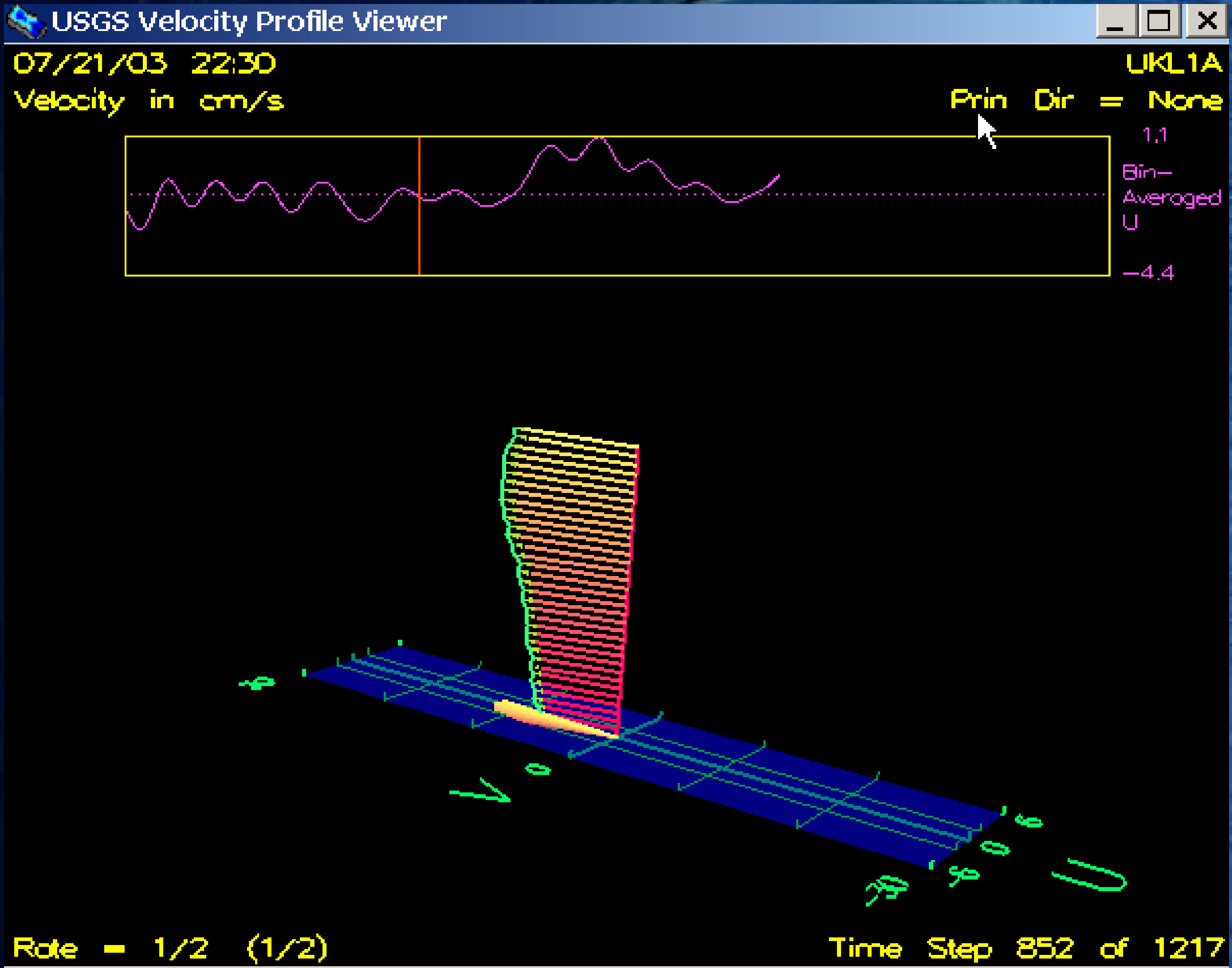
Sampling rate = 30.0 min

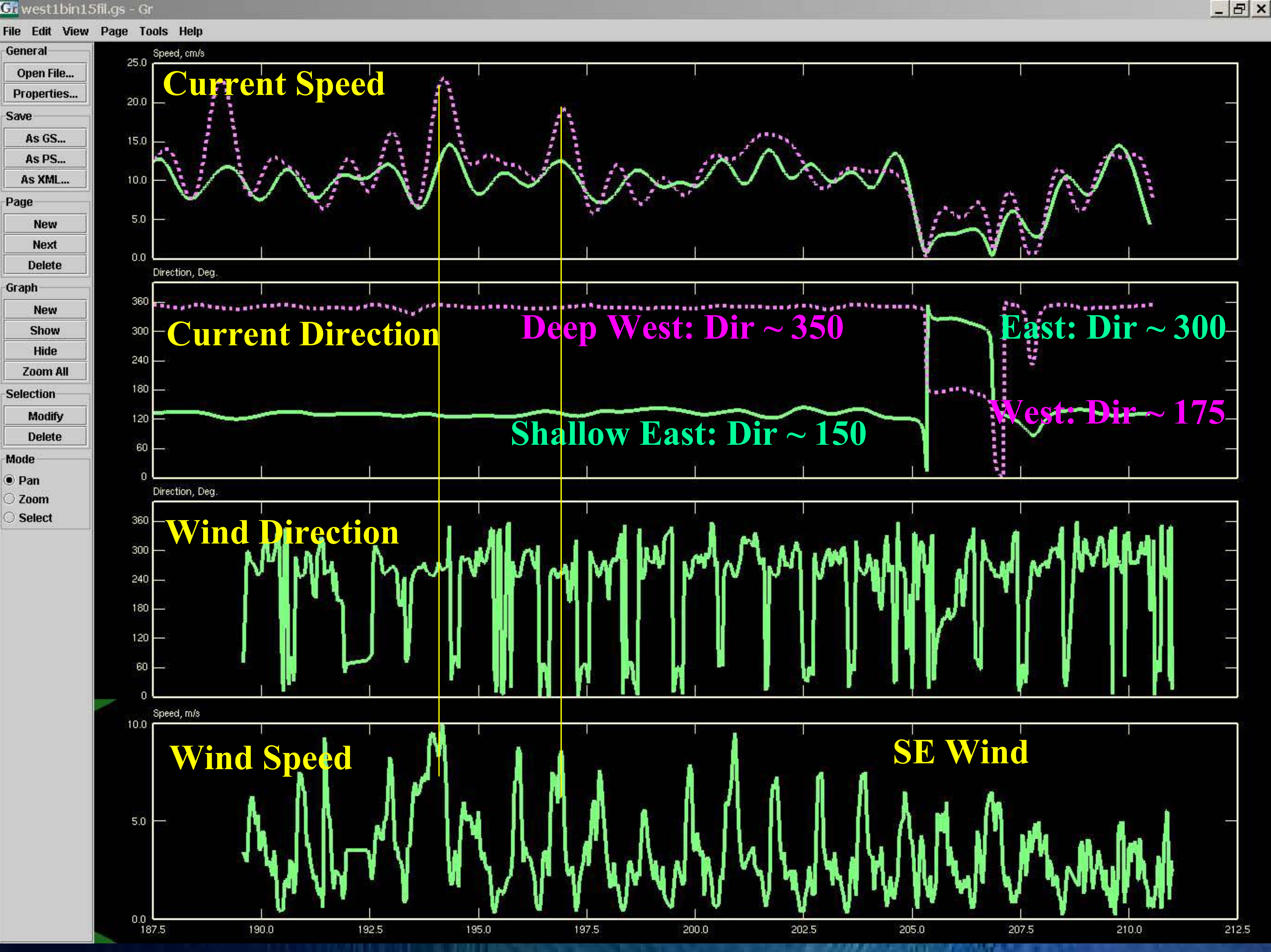
Total bins = 12

Wind Speed and Direction Time-Series

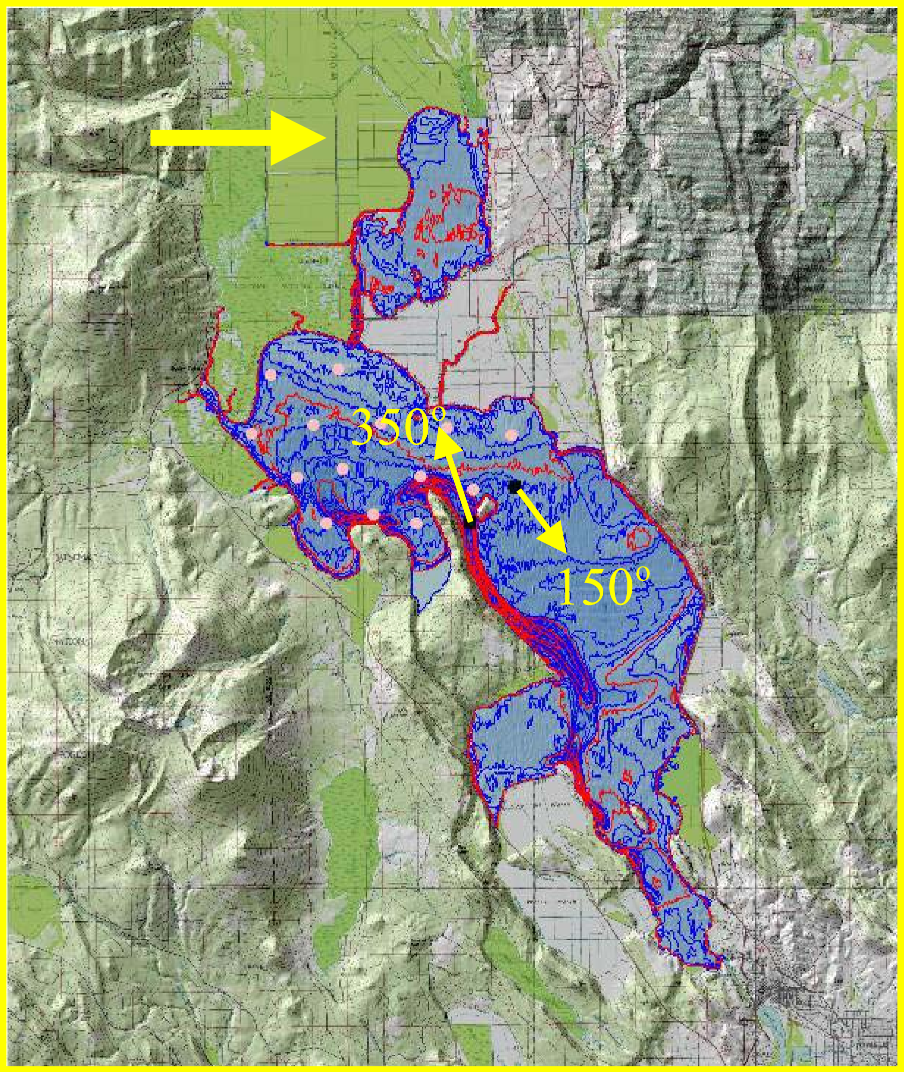


Filtered 3D ADCP Time-Series

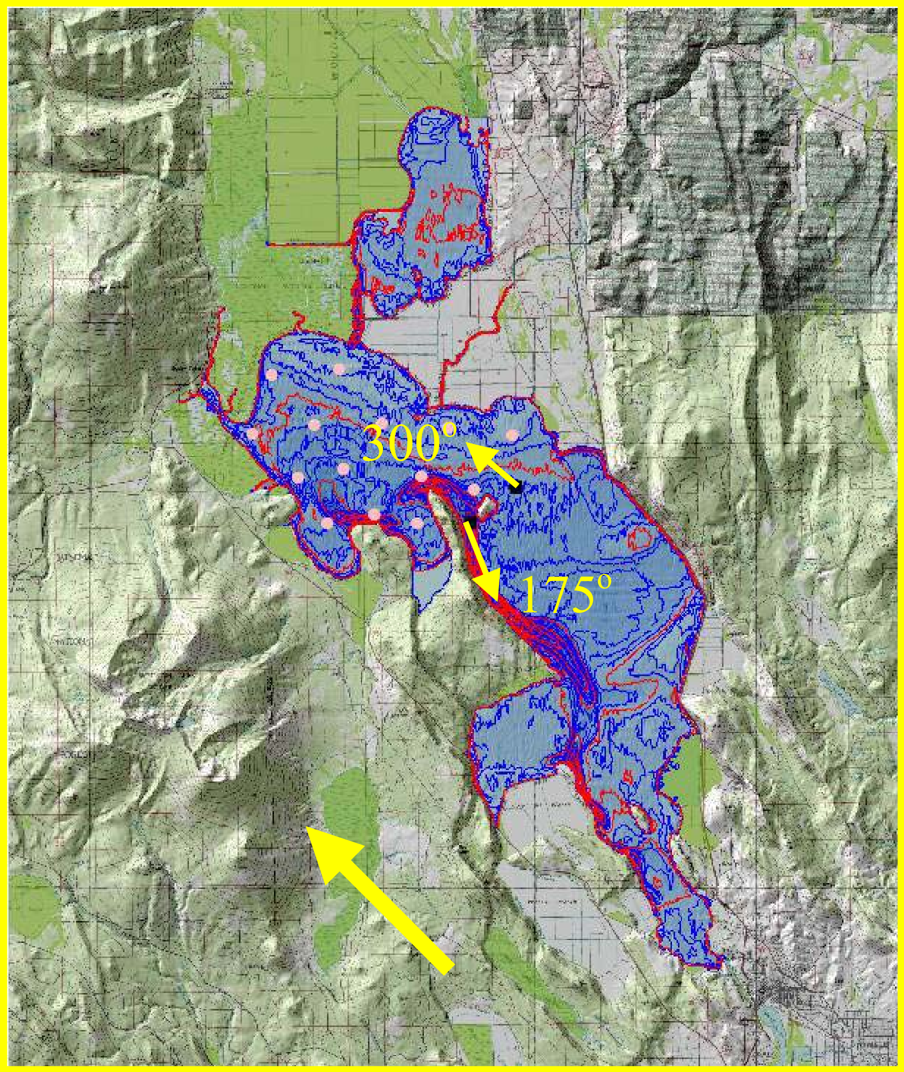



west1bin15fil.gs - Gr
File Edit View Page Tools Help
General
Open File...
Properties...
Save
As GS...
As PS...
As XML...
Page
New
Next
Delete
Graph
New
Show
Hide
Zoom All
Selection
Modify
Delete
Mode
Pan
Zoom
Select
Speed, cm/s
Current Speed
Direction, Deg.
Current Direction
Deep West: Dir ~ 350
East: Dir ~ 300
West: Dir ~ 175
Shallow East: Dir ~ 150
Direction, Deg.
Wind Direction
Speed, m/s
Wind Speed
SE Wind
187.5
190.0
192.5
195.0
197.5
200.0
202.5
205.0
207.5
210.0
212.5

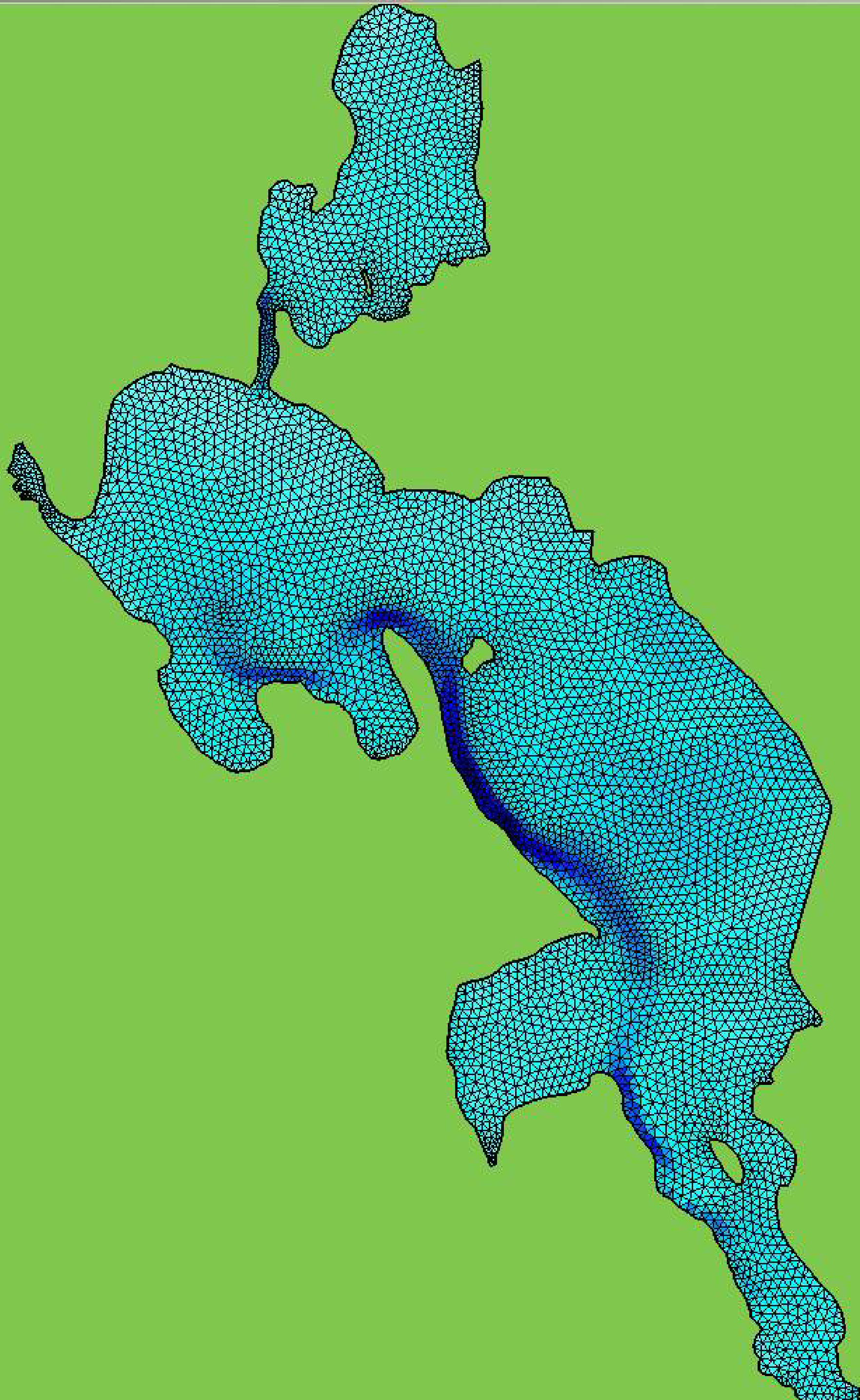
Synopsis of Wind-driven Circulation



Prevailing West Wind



SE Wind



Unstructured Grid Model:

Upper Klamath Lake and Agency Lake:

nv = 4712

ne = 8550

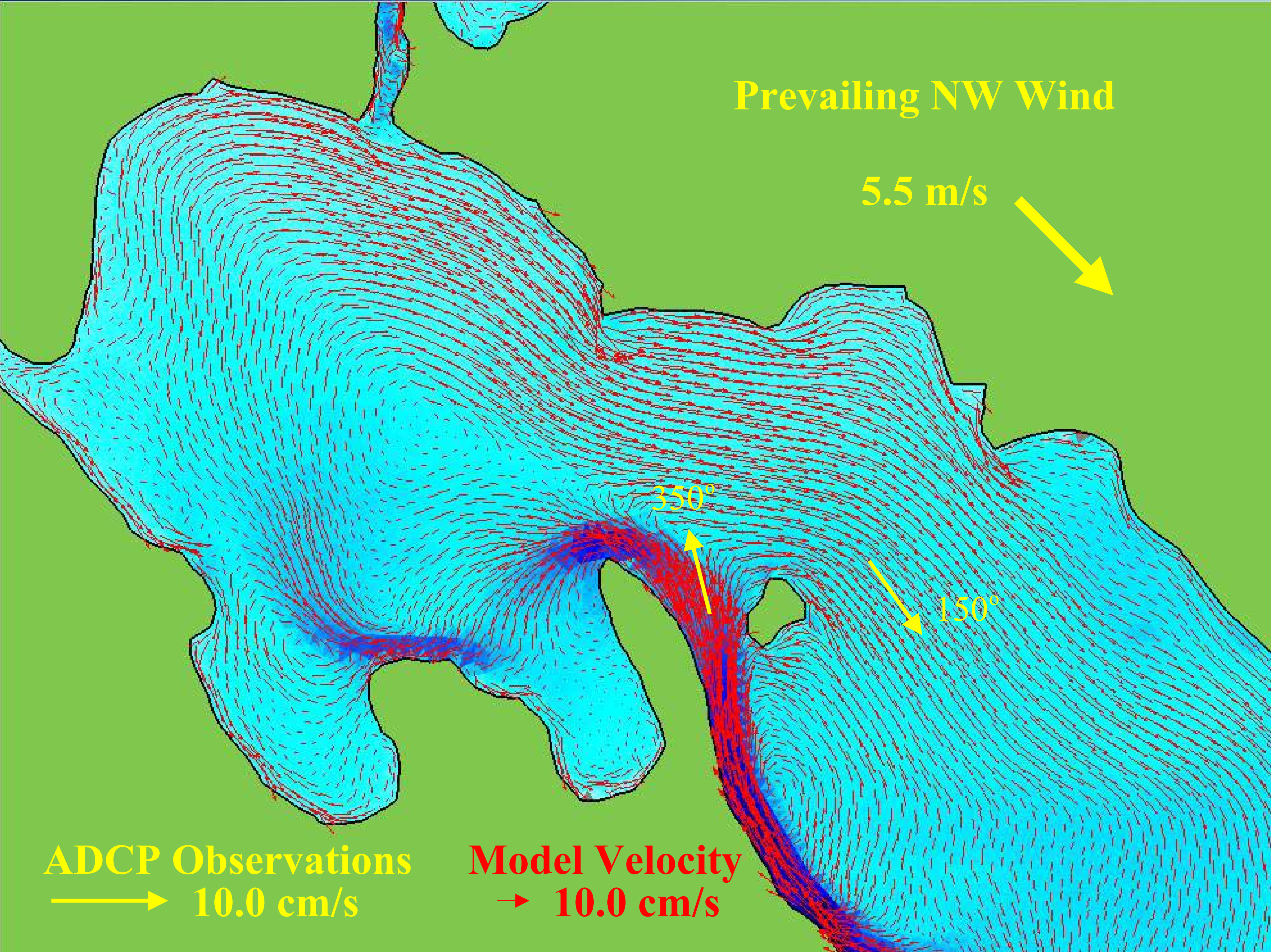
nk = 22

n3s = 82992

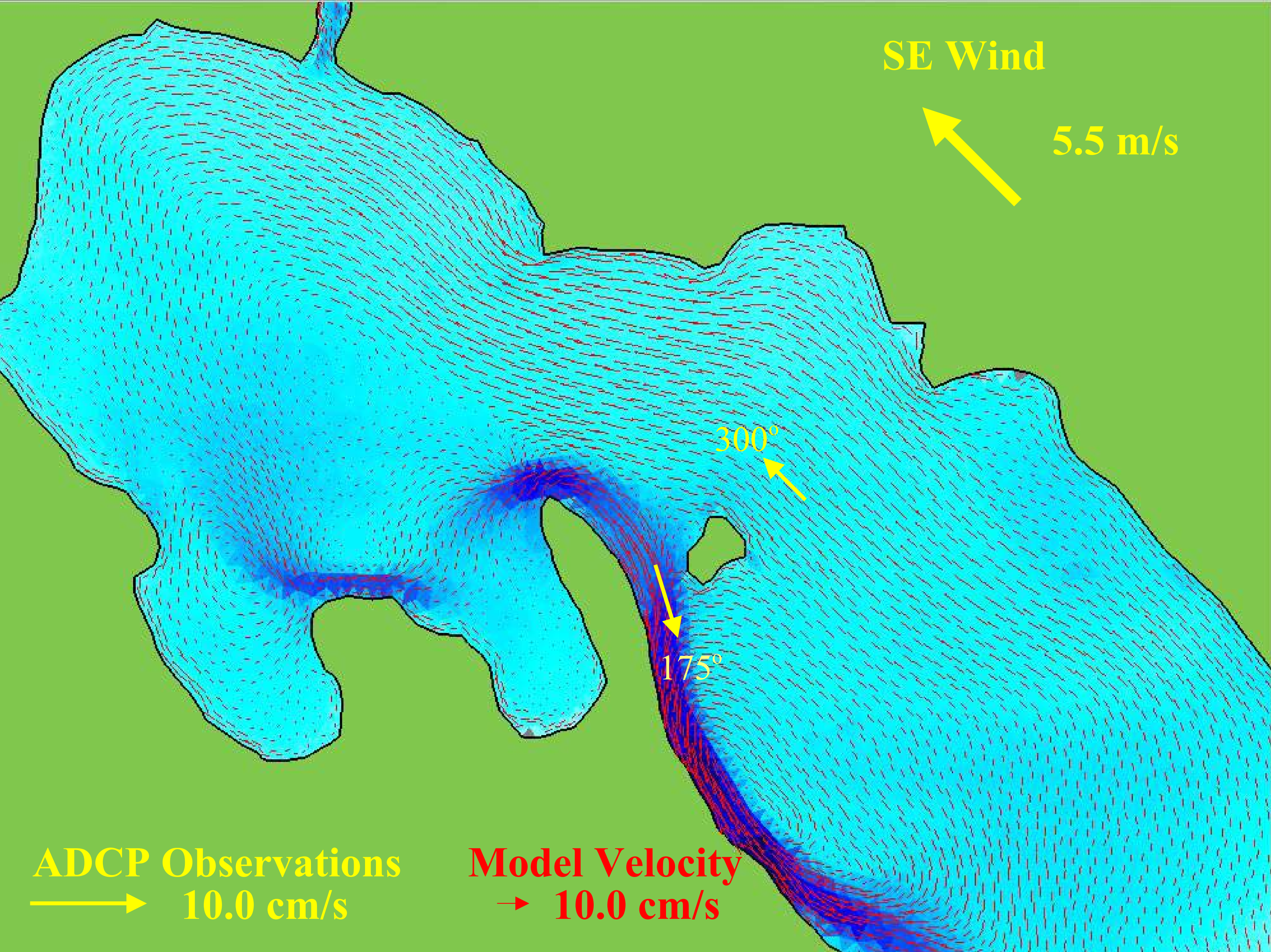
Side length 40 to 250 m

Grids are boundary fitting

Fine resolution grids for high spatial variability.



Prevailing NW Wind
5.5 m/s
350°
150°
ADCP Observations
10.0 cm/s
Model Velocity
10.0 cm/s


SE Wind
5.5 m/s
300°
175°
ADCP Observations
10.0 cm/s
Model Velocity
10.0 cm/s

Field Data:

Observed Wind

Deep ADCP (West)

Shallow ADCP (East)

Williamson River Inflow

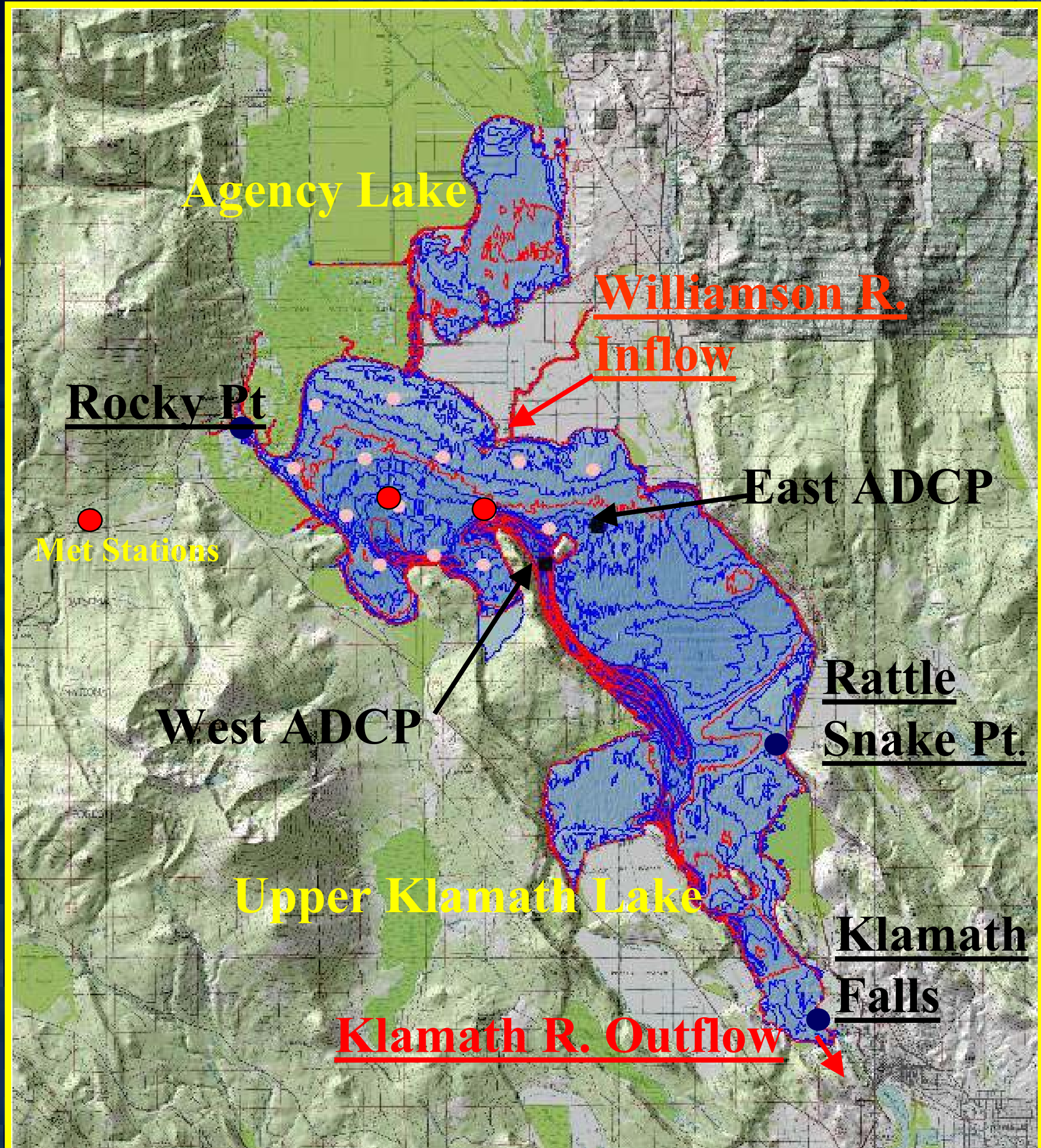
Klamath R. Outflow

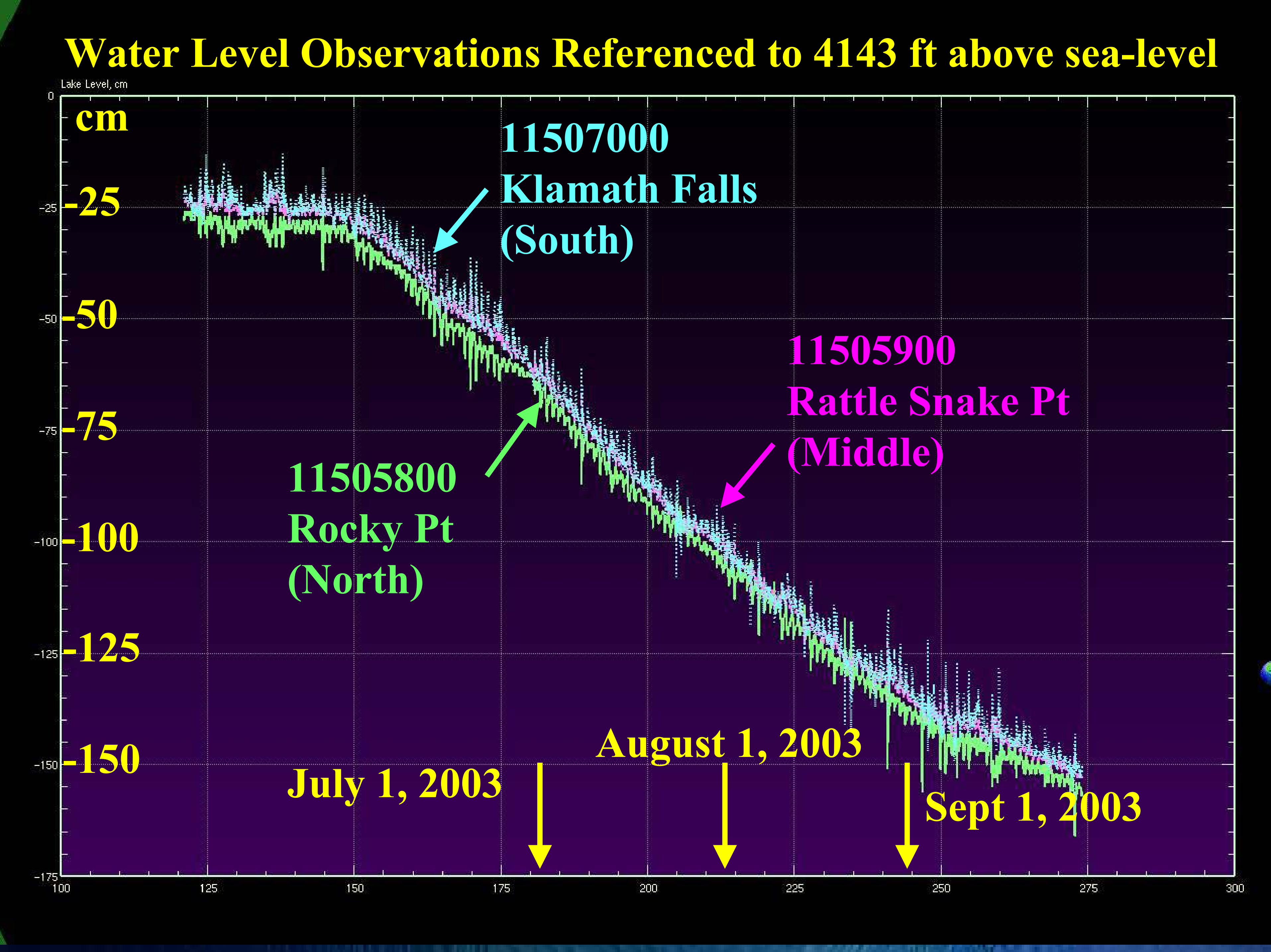
Water levels at

Rocky Pt

Rattle Snake Pt.

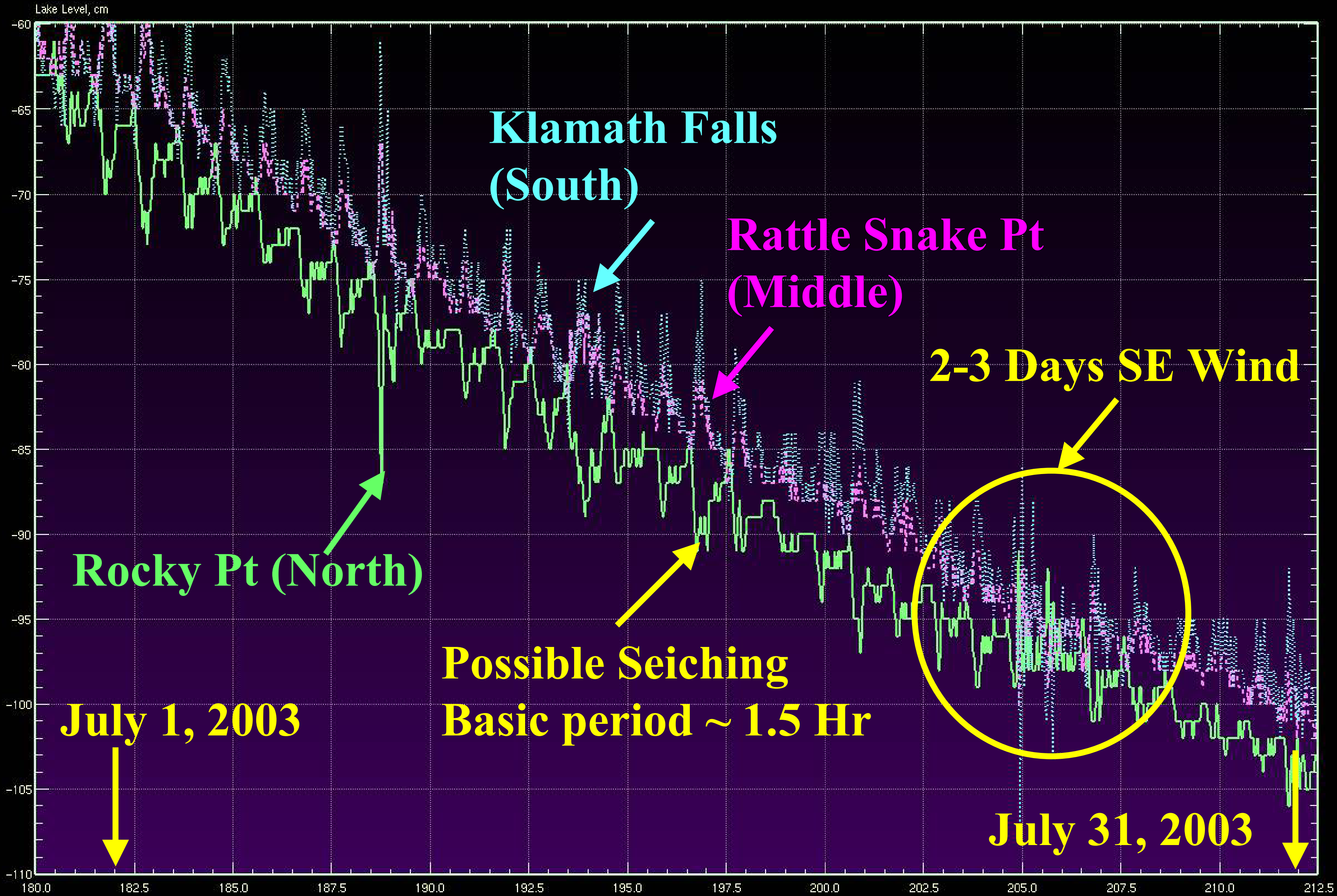
Klamath Falls



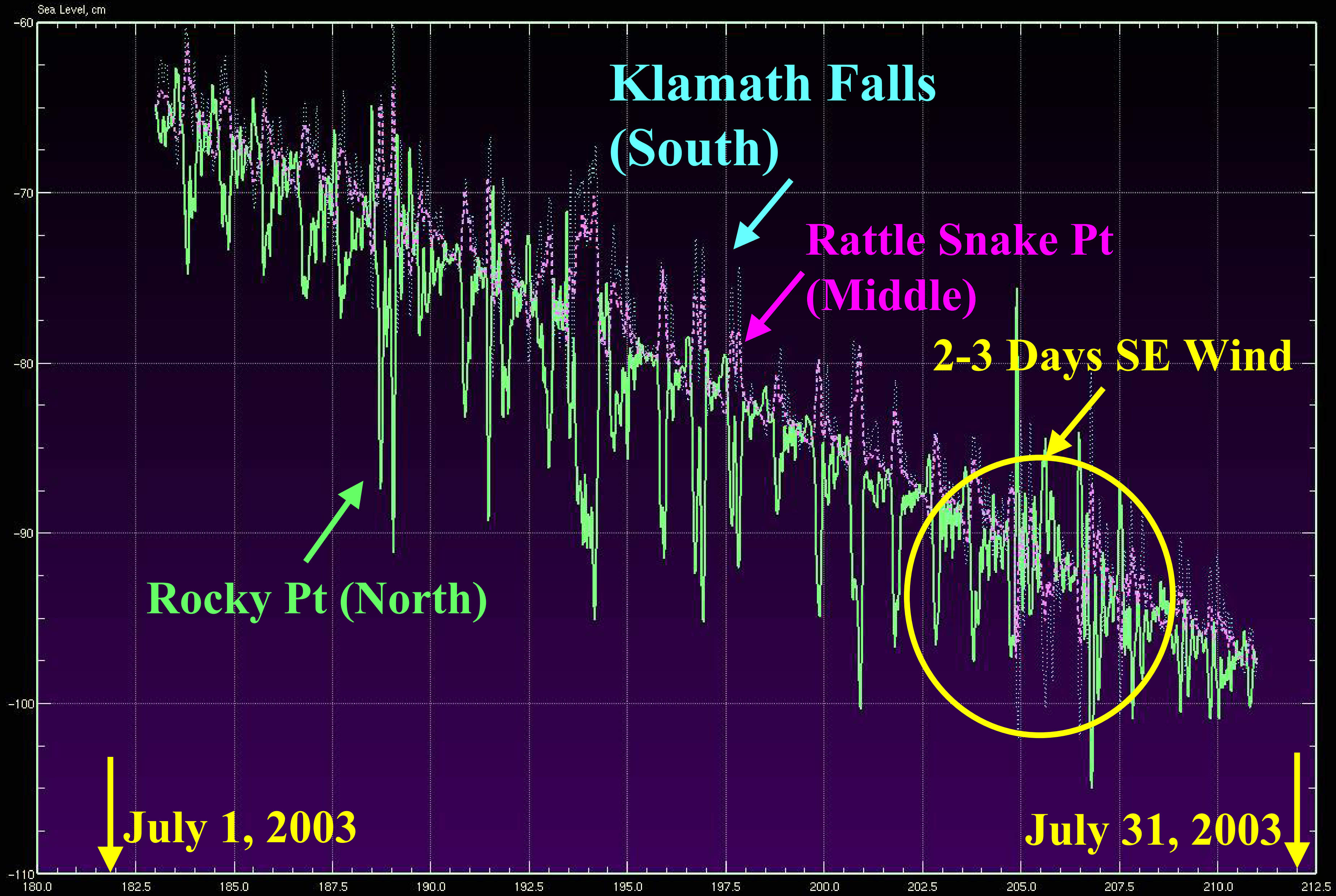


Water Level Observations Referenced to 4143 ft above sea-level
Lake Level, cm
cm
11507000
Klamath Falls
(South)
11505900
Rattle Snake Pt
(Middle)
11505800
Rocky Pt
(North)
-25
-50
-75
-100
-125
-150
July 1, 2003
August 1, 2003
Sept 1, 2003
0
-25
-50
-75
-100
-125
-150
-175
100
125
150
175
200
225
250
275
300

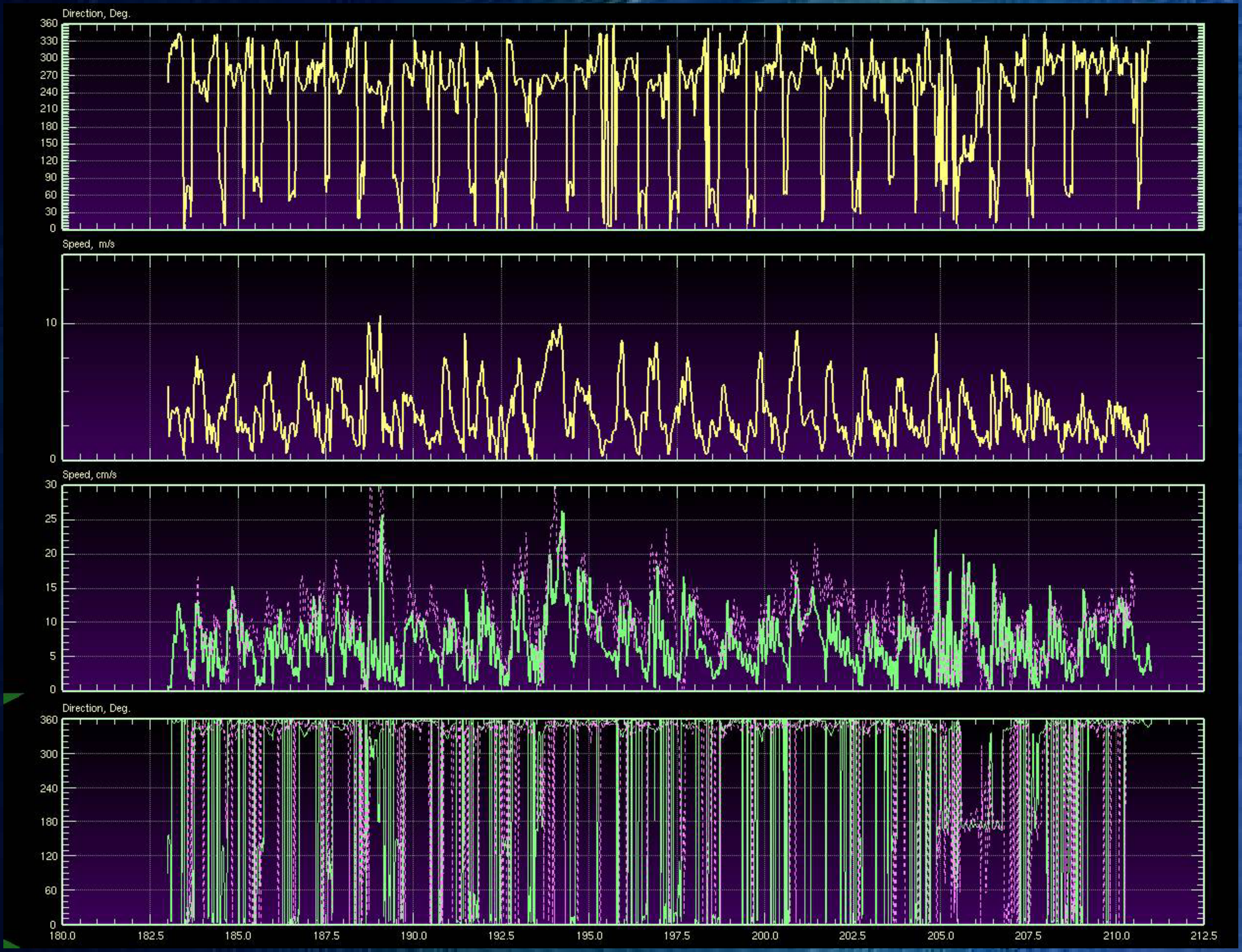
Water Level Observations Referenced to 4143 ft above sea-level

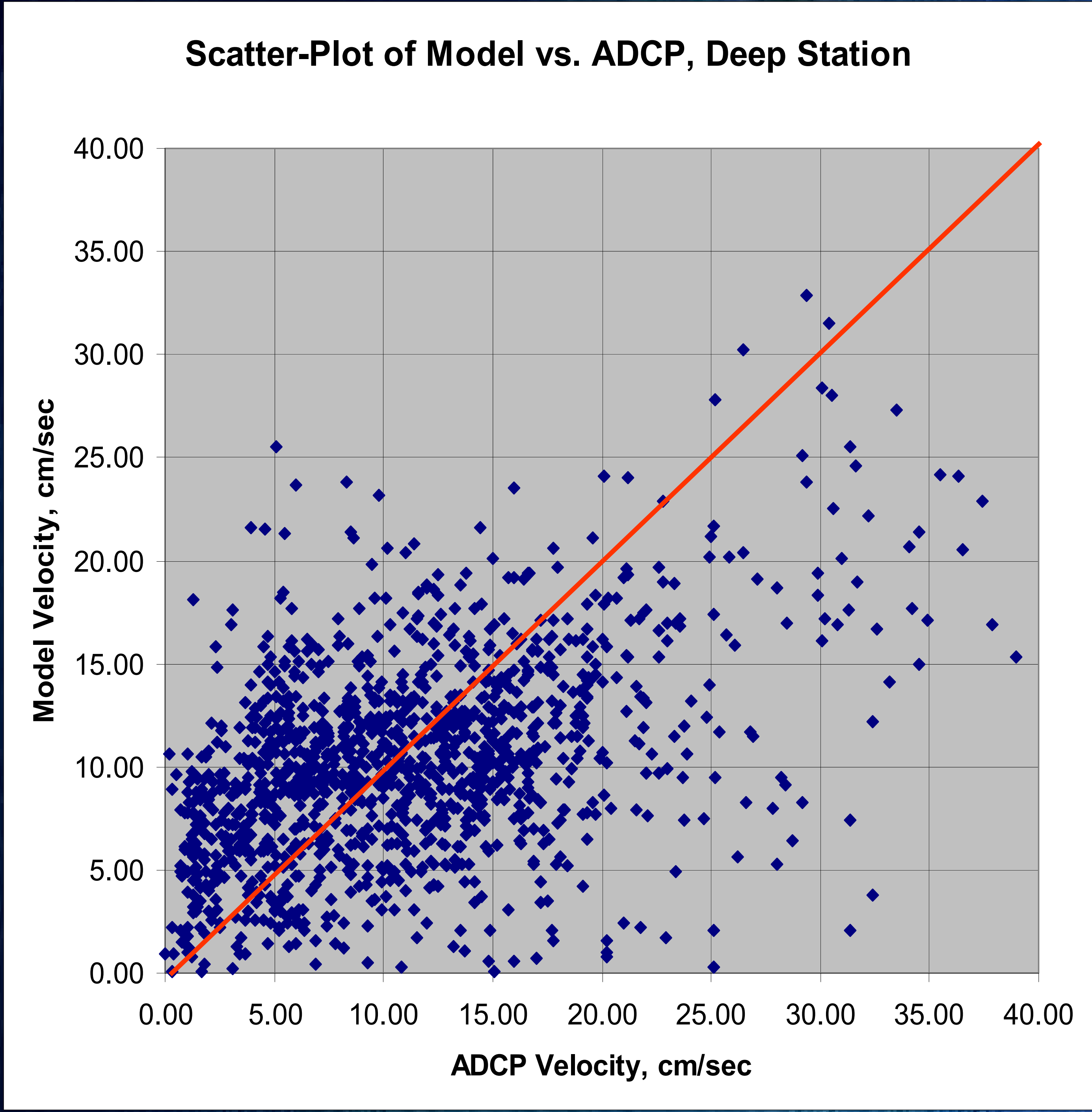


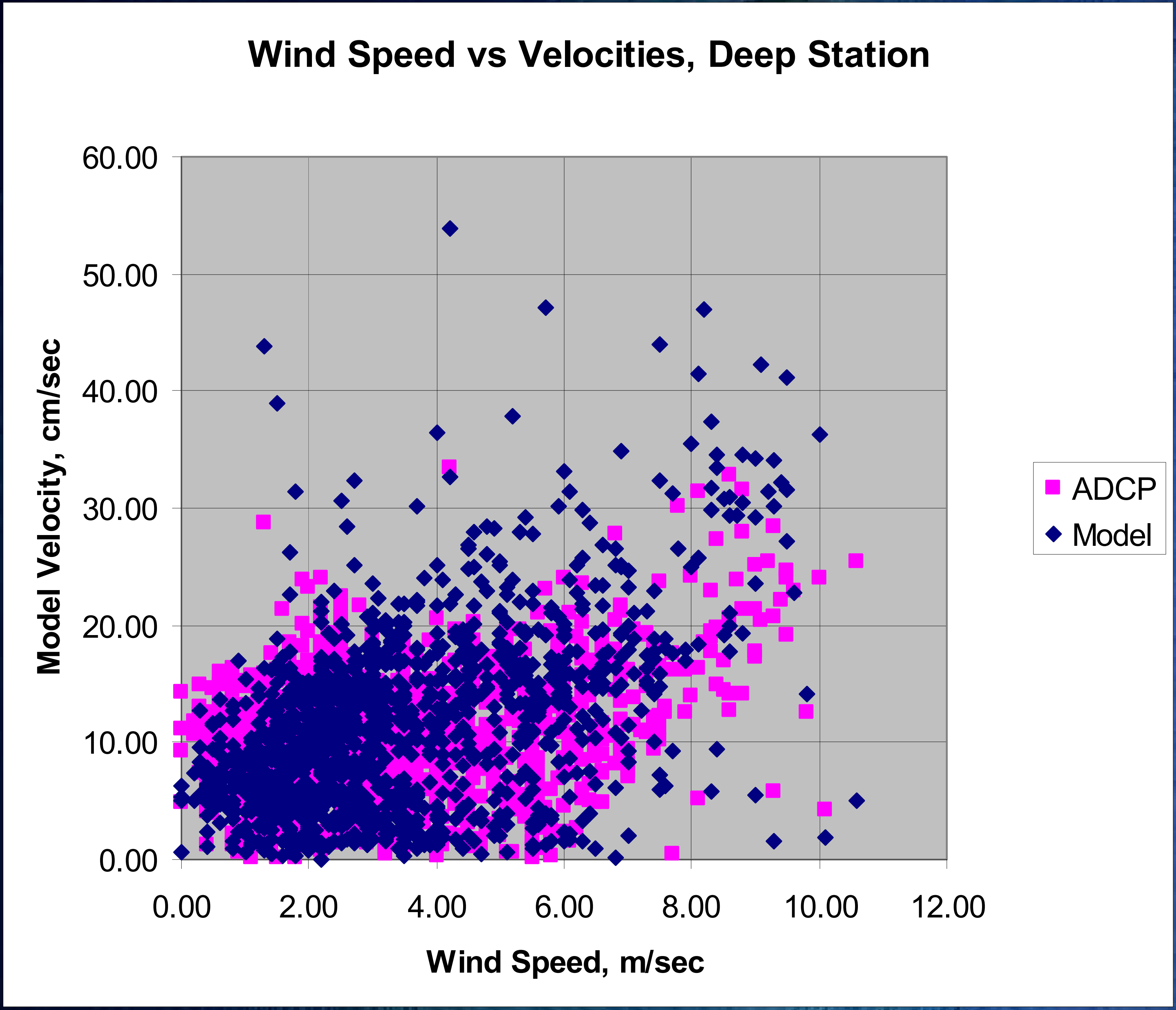
Model Simulated Water Level Variations



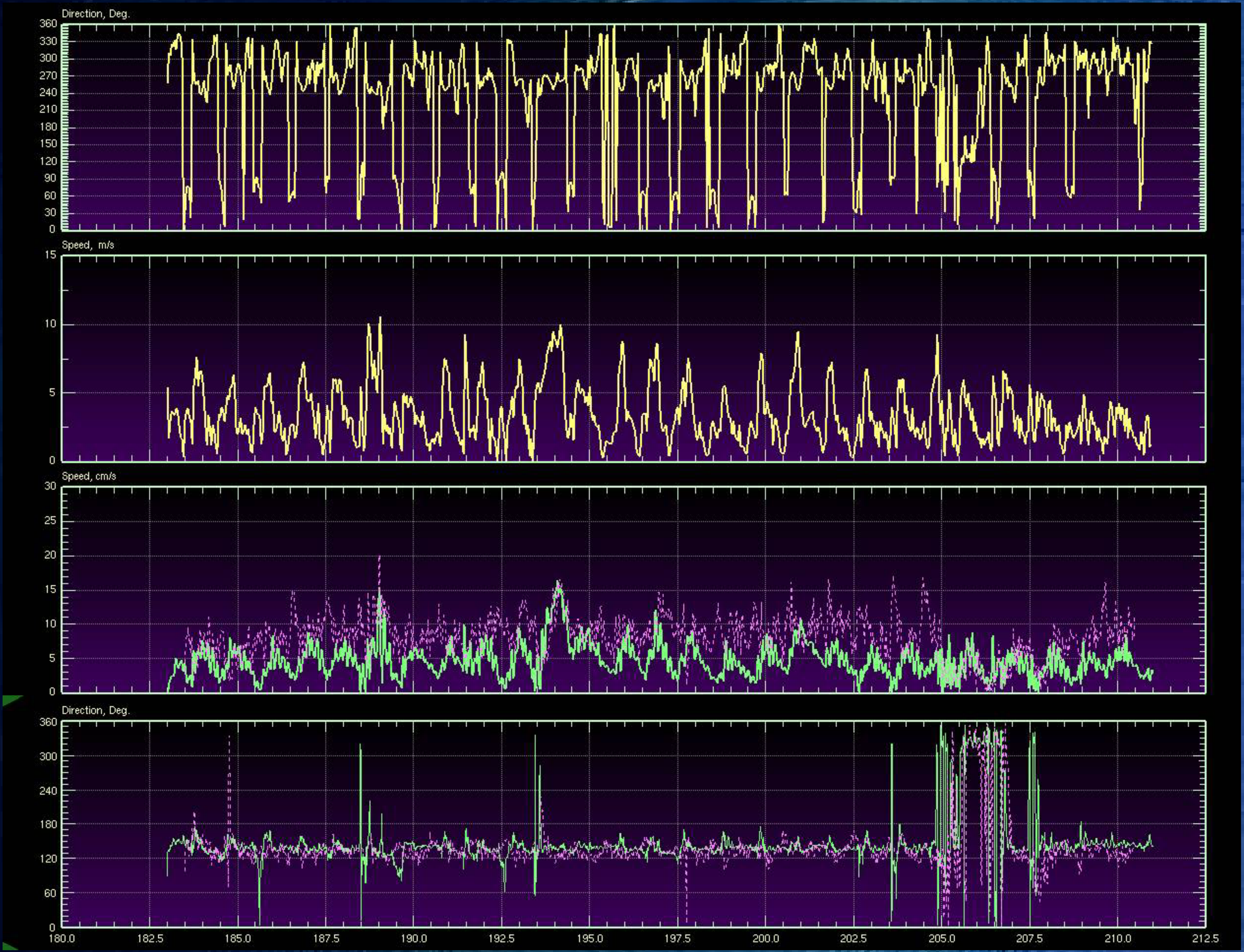
Model Results vs. ADCP Observations at Deep (West) Station

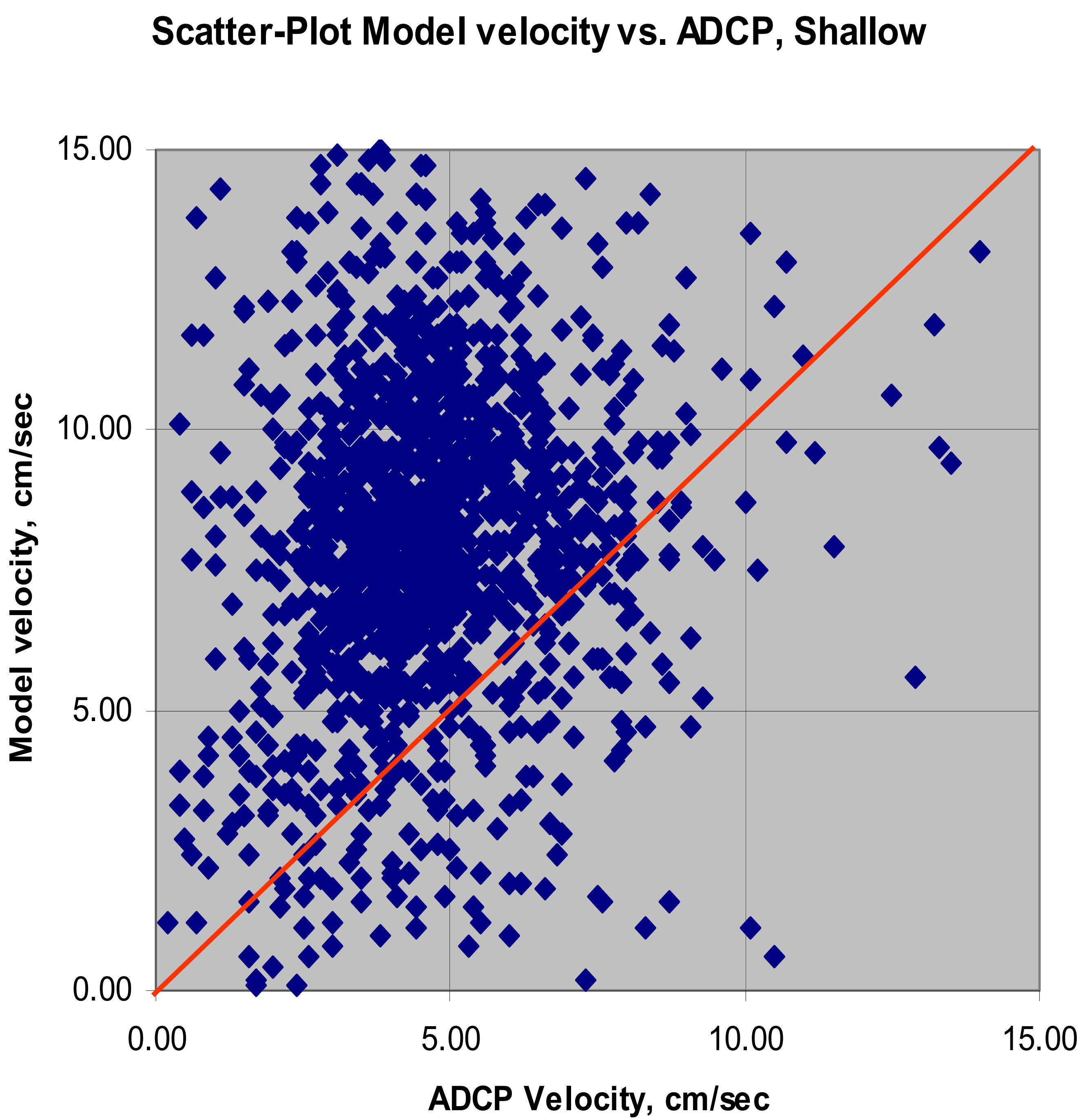



Scatter-Plot of Model vs. ADCP, Deep Station
Model Velocity, cm/sec
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
0.00
5.00
10.00
15.00
20.00
25.00
30.00
35.00
40.00
ADCP Velocity, cm/sec

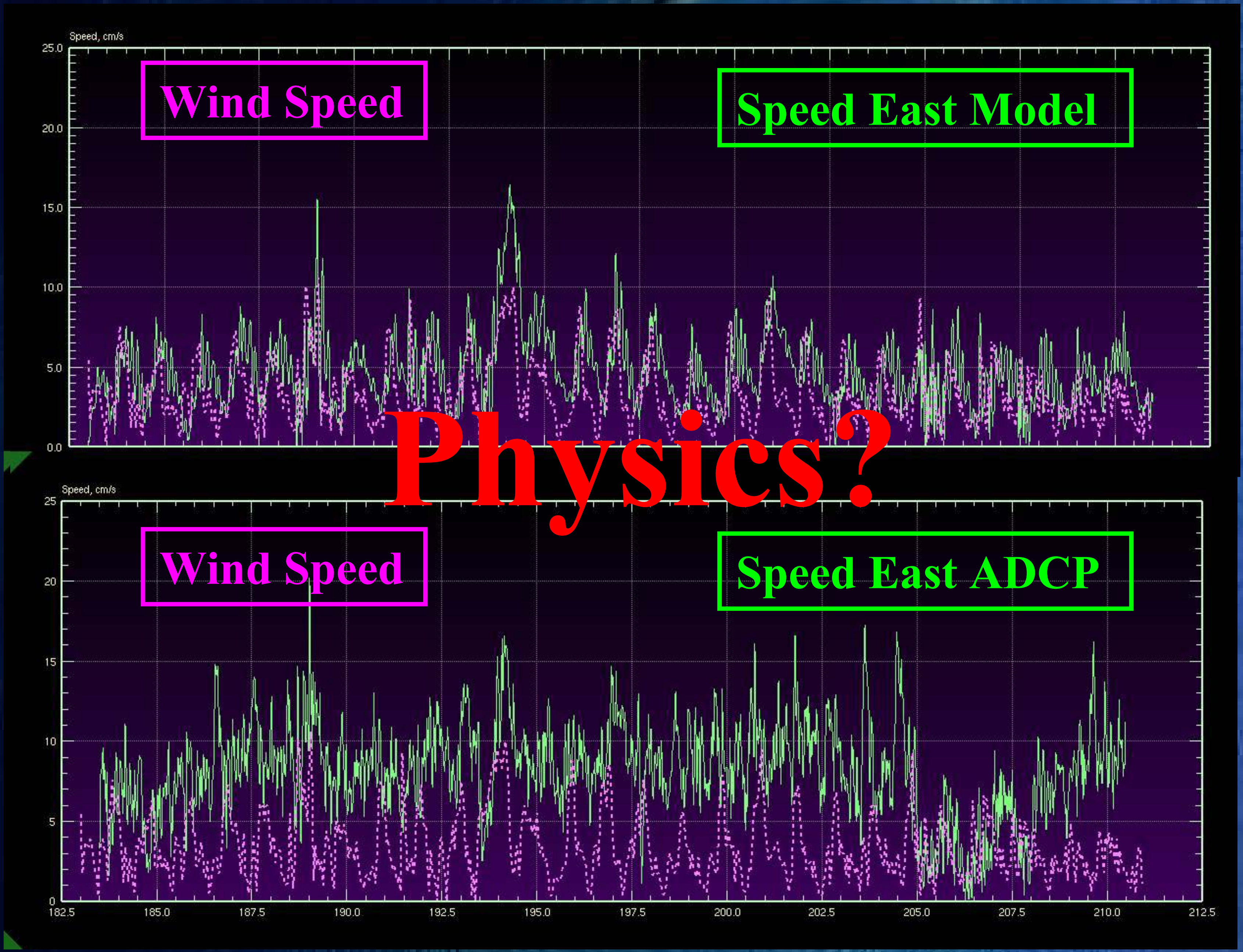

Wind Speed vs Velocities, Deep Station
Model Velocity, cm/sec
60.00
50.00
40.00
30.00
20.00
10.00
0.00
0.00
2.00
4.00
6.00
8.00
10.00
12.00
Wind Speed, m/sec
ADCP
Model

Model Results vs. ADCP Observations at Shallow (East) Station

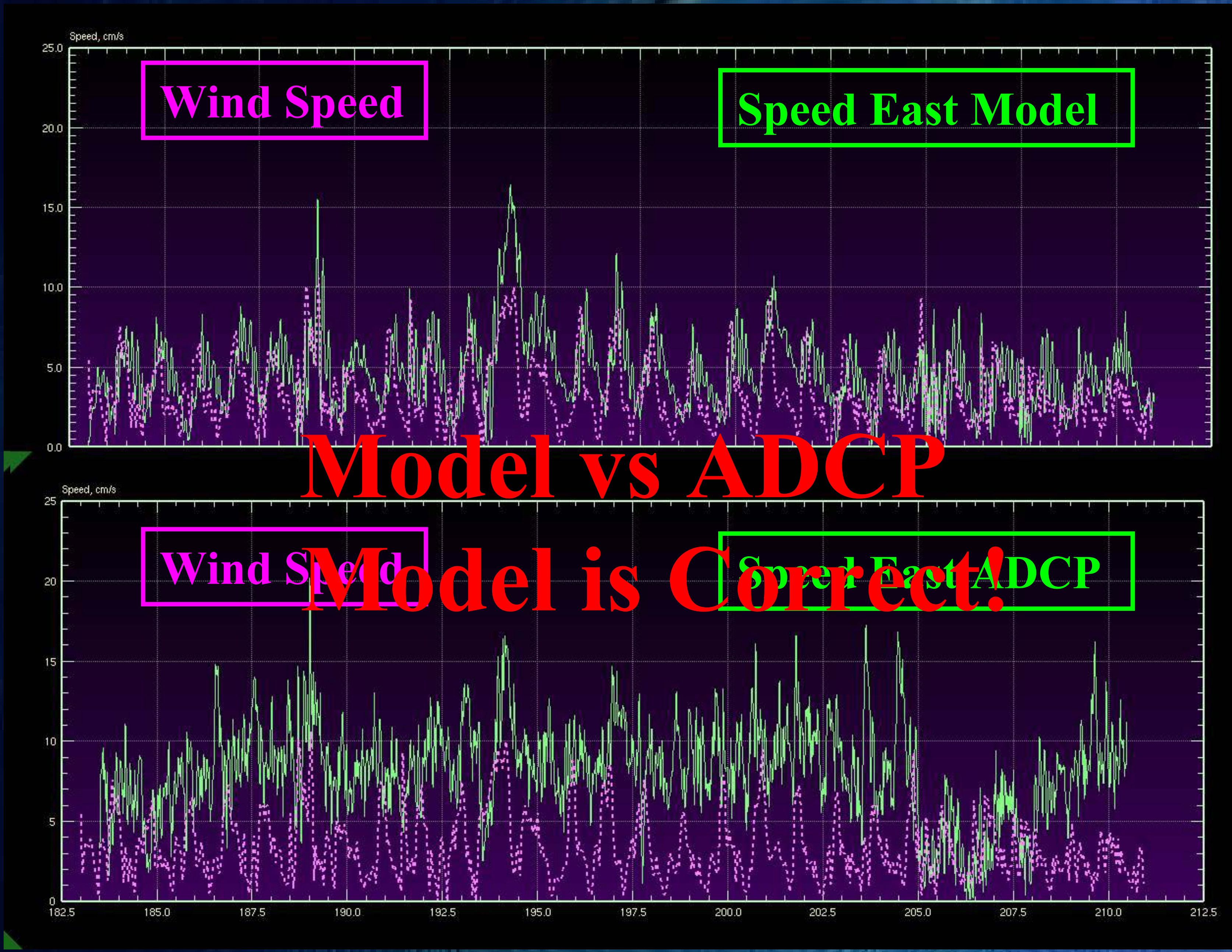



Scatter-Plot Model velocity vs. ADCP, Shallow
15.00
10.00
5.00
0.00
0.00
5.00
10.00
15.00
Model velocity, cm/sec
ADCP Velocity, cm/sec

Correlations with wind speed



Correlations with wind speed



Take Home Message:

Field Data Do not Necessarily Represent the Truth.

Field Data Must be Consistent with the Correct Physics!

There might be hidden messages in the data!

Conclusion

- Circulation in Upper Klamath Lake is shown to be <u>completely controlled by wind.</u>
- The ADCP data at a deep station is reproduced reasonably well; at the shallow station, data are shown to be suspect.
- Discrepancies are due to the uncertainty in wind records and the assumption of uniform wind used to drive the model
- Only to improve the quality of inputs can improve model results

Conclusion and Summary

Through a large number of field applications, the UnTRIM model has been shown to be a robust, flexible, and computationally efficient model for studying Environmental Flows.

謝謝！

☺

Thank You!